Reality as Equivalence: Univalent Relational Dynamics, Category Theory, Homotopy Type Theory, and an Equivalence-First Foundation for Mathematical Physics

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Modern physics is usually formulated in terms of states, fields, particles, and trajectories, with equivalence treated as a secondary relation imposed afterward by symmetry, gauge freedom, or coordinate choice. This paper reverses that hierarchy. It proposes Univalent Relational Dynamics, an equivalence-first mathematical physics in which states are local presentations, physical identity is constituted by admissible equivalence, and dynamics describes the transport, deformation, composition, and obstruction of those equivalences. Category theory supplies the language of objects and processes, while homotopy type theory supplies identity as path, higher identity as paths between paths, and univalence as the principle that structural equivalence can serve as physical sameness. From this foundation, particles become stable defects in coherence, charges become representations of automorphism structure, entanglement becomes nonfactorizable identity, measurement becomes truncation of higher paths, entropy becomes multiplicity of equivalent presentation, and spacetime emerges from the cost of coherent transport. A finite genetic-code model motivates the program by showing how many presentations can collapse into fewer meanings while retaining nontrivial adjacency, redundancy, and topology. The result is a testable framework for deriving geometry, matter, and law from equivalence itself.

Keywords: equivalence-first physics, Univalent Relational Dynamics, category theory, homotopy type theory, univalence axiom, mathematical physics, higher category theory, infinity-groupoids, identity types, relational ontology, gauge equivalence, emergent spacetime, quantum foundations, categorical quantum mechanics, homotopy physics, coherence, higher symmetries, topological defects, automorphism groups, entanglement, measurement, truncation, entropy, moduli spaces, genetic code topology.

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Outline

1. From State-Based Physics to Equivalence-First Reality

1.1 The conventional ontology of states

Introduce the standard view in which physical systems possess states, observables act on those states, and dynamical laws describe transitions or trajectories through a state space.

1.2 Equivalence as a secondary construction

Examine how conventional theories introduce equivalence through gauge transformations, coordinate changes, symmetry operations, indistinguishable particles, and alternative mathematical representations.

1.3 The ontological reversal

Propose that states are not fundamental entities but local presentations of a deeper relational structure. Equivalences, transformations, and their coherence relations are placed before objects in the explanatory hierarchy.

1.4 The foundational statement

Develop the central claim:

Reality is not made of states; it is made of equivalence among states.

Clarify that this does not deny observations or configurations. It denies that an isolated configuration possesses complete physical meaning independently of the transformations under which it remains identifiable.

1.5 Persistence rather than substance

Define physical existence operationally as persistence across admissible transformation. An entity exists to the extent that a coherent criterion identifies it through changes of coordinates, representation, environment, and time.

1.6 Scope of Univalent Relational Dynamics

Present Univalent Relational Dynamics as a proposed mathematical framework rather than an established replacement for current physics. State that the objective is to construct definitions, equations, limiting cases, computational models, and potentially falsifiable consequences.

2. Category Theory, Homotopy Type Theory, and Physical Identity

2.1 Categories of physical processes

Introduce objects as local presentations and morphisms as transformations, observations, interactions, or processes. Emphasize composition and identity morphisms as the minimal architecture of process-based physics.

2.2 From categories to higher categories

Explain why ordinary categories are insufficient when transformations between transformations have physical significance. Introduce 2-morphisms and higher morphisms as coherence data.

2.3 Infinity-groupoids and reversible structure

Describe an infinity-groupoid as a system in which objects are connected by invertible paths, paths are connected by higher paths, and this hierarchy continues without a fixed upper level.

2.4 Identity as path

Introduce the homotopy-type-theoretic interpretation of identity. For presentations x and y, an identity witness is not merely a Boolean declaration but a structured path from x to y.

2.5 Univalence as a physical principle

State the proposed physical form of univalence:

Id_phys(x,y) <-> Equiv_phys(x,y).

Interpret this as the principle that physically admissible structural equivalence constitutes physical identity rather than merely indicating similarity.

2.6 Equivalence spaces rather than equivalence predicates

Replace the binary question of whether two presentations are equivalent with a space Equiv_phys(x,y) containing potentially many distinct equivalences, deformations, symmetries, and higher relations.

2.7 Identity without primitive objects

Explore the possibility that objects are recoverable from their patterns of incoming and outgoing relations, automorphisms, and compositional behavior, connecting the physical proposal with categorical reconstruction principles.

3. The Axioms of Univalent Relational Dynamics

3.1 The presentation axiom

Every observed state is a presentation relative to an observational, mathematical, or experimental context. No presentation exhausts the underlying physical identity.

3.2 The univalent identity axiom

Two presentations represent the same physical entity precisely to the degree that an admissible equivalence connects them.

3.3 The higher-coherence axiom

Equivalences between equivalences, and higher relations among those equivalences, possess physical content whenever they affect composition, transport, obstruction, or observation.

3.4 The observable-descent axiom

A valid physical observable must descend through equivalence. If x and y are physically equivalent, an observable must assign equal or appropriately equivalent results:

$x \sim= y$ implies $O(x) \sim= O(y)$.

3.5 The compositionality axiom

Physical processes compose. A process from x to y followed by a process from y to z defines a process from x to z, subject to coherent associativity.

3.6 The local-to-global axiom

Globally meaningful physical structures arise by coherently gluing compatible local presentations and local equivalences. Global obstruction is itself a physical phenomenon.

3.7 The dynamical-coherence axiom

Dynamics governs the formation, transport, deformation, stabilization, and failure of equivalence structures.

3.8 The finite-observation axiom

Actual observations access truncated portions of higher relational structure. Classical outcomes arise when higher distinctions become experimentally inaccessible.

4. Presentation Spaces, Moduli Structures, and Homotopy Quotients

4.1 The space of presentations

Let P denote the collection of mathematical or physical presentations available within a theory. Explain why P generally contains redundant descriptions.

4.2 Equivalence relations and groupoid structure

Replace a simple equivalence relation on P with a groupoid recording explicit transformations, stabilizers, automorphisms, and composition laws.

4.3 The moduli space of physical meanings

Introduce a projection

q: P -> M,

where M represents physical meanings or equivalence classes of presentation.

4.4 Why an ordinary quotient is insufficient

Show that the quotient set P/~ forgets how presentations are related, how many transformations connect them, and what automorphisms preserve each presentation.

4.5 The homotopy quotient

Introduce the richer construction

M = P // E,

where E represents the admissible equivalence structure. Explain how the homotopy quotient retains stabilizers, loops, and higher coherence.

4.6 Fibers of representation

Interpret q^-1(m) as the space of presentations realizing one physical meaning m. Discuss fiber size, topology, symmetry, and dimensionality.

4.7 Automorphism groups

Define Aut(x) as the self-equivalences of a presentation x. Explain how automorphism structure can encode internal symmetry, conserved labels, and physical degeneracy.

4.8 Homotopy groups as physical invariants

Interpret pi_0 as disconnected sectors, pi_1 as symmetry loops or holonomy, pi_2 as relations among loops, and higher pi_n as generalized topological structures.

5. Dynamics as Transport and Extremization of Coherence

5.1 From trajectories to transformations

Replace the primitive trajectory x(t) with a composable family of transformations carrying one local presentation into another.

5.2 Identity transport

Define transport along a path gamma as

T_gamma: F_x -> F_y.

Interpret T_gamma as the rule for carrying physical identity between contexts.

5.3 Coherence around loops

For a closed path, compare T_loop with the identity transformation. Nontrivial return transport indicates curvature, holonomy, phase, or topological obstruction.

5.4 Coherence cost

Introduce a functional C(e) assigning a cost to an equivalence, transport map, or deformation. Low-cost transformations define physical proximity.

5.5 A relational action principle

Propose a schematic action

S = sum_edges C(T_ij) + lambda sum_loops K(T_loop,id) + mu sum_vertices B(x_i),

where C measures transport cost, K measures failure of loop closure, and B measures local incompatibility or boundary cost.

5.6 The principle of maximal coherent equivalence

Propose that physically realized structures extremize the preservation of equivalence while minimizing the cost required to sustain it.

5.7 Equivalence geodesics

Define an equivalence geodesic as a transformation path minimizing coherence cost between two physical contexts.

5.8 Continuum limits

Outline how a discrete relational action may produce continuum expressions involving covariant derivatives, curvature, potentials, and volume measures.

5.9 Conservation from coherent symmetry

Reinterpret conservation laws as invariants preserved by families of equivalences and their compositional structure.

6. Emergent Geometry, Spacetime, and Gravitation

6.1 Relational proximity

Propose that two contexts are physically near when a low-cost coherent identification connects them.

6.2 Distance from equivalence cost

Define a candidate emergent distance:

$d(x,y) = \inf_e C(e)$,

where the infimum is taken over admissible transformations from x to y.

6.3 Topology before metric

Construct neighborhoods from the existence and density of low-cost equivalences before introducing numerical distance.

6.4 Metric structure from transport

Investigate the conditions under which coherence cost satisfies positivity, symmetry, separation, and triangle-like composition properties.

6.5 Curvature as failed identity closure

Interpret curvature as the obstruction encountered when identity is transported around a loop and fails to return trivially.

6.6 Gravitation as coherence geometry

Develop the hypothesis that gravitational geometry is the macroscopic expression of position-dependent identity transport.

6.7 Equivalence and the equivalence principle

Compare the proposed framework with the physical equivalence of local acceleration and gravitation, while distinguishing univalent equivalence from the traditional equivalence principle.

6.8 Spacetime dimension

Ask whether effective dimension can emerge from the growth rate, connectivity, or embedding requirements of an underlying equivalence complex.

6.9 Lorentzian structure and causal order

Investigate how causal distinction and Lorentzian signature might arise when some morphisms are directed and noninvertible while spatial identifications remain locally reversible.

6.10 Background independence

Explain why an equivalence-generated geometry need not assume a preexisting spacetime arena.

7. Matter as Stable Coherence, Defect, and Representation

7.1 Objects as persistent relational patterns

Define a physical object as a pattern of equivalences that can be transported while retaining a coherent identity type.

7.2 Particles as topological defects

Propose that particle-like entities are localized obstructions to globally trivializing an equivalence field.

7.3 Mass as equivalence inertia

Interpret mass as the action cost required to preserve and transport a coherence pattern:

C(E,gamma) approximately equals m times the path length in an appropriate low-energy limit.

7.4 Rest mass and internal coherence

Explore whether rest mass measures coherence maintenance independent of translational motion.

7.5 Massless propagation

Characterize a massless excitation as an equivalence pattern capable of propagation without an intrinsic rest-frame coherence cost.

7.6 Charge as automorphism response

Define charge by how a physical identity transforms under local self-equivalences and representations of Aut(x).

7.7 Internal symmetries

Investigate whether familiar gauge groups can arise as automorphism groups of physically admissible local identity structures.

7.8 Antiparticles and duality

Explore antiparticles as dual, inverse, oppositely oriented, or conjugate coherence structures.

7.9 Composite matter

Use categorical composition, tensor products, limits, colimits, and gluing to describe the formation of composite systems.

7.10 Interaction as coherence exchange

Reinterpret interactions as changes in how identity structures factor, compose, or obstruct one another.

8. Quantum Theory as an Integral Over Equivalences

8.1 From state vectors to relational amplitudes

Recast quantum amplitudes as assignments to transformations and equivalence classes rather than primitive ontological states.

8.2 The equivalence path integral

Introduce the schematic amplitude

K([x],[y]) = integral over e:x->y of exp(iS[e]/hbar) De.

Interpret the integration domain as a space of admissible transformations connecting physical meanings.

8.3 Interference among identifications

Describe quantum interference as constructive or destructive combination among distinct coherent ways of relating preparation and outcome.

8.4 Superposition as unresolved equivalence structure

Interpret superposition as the retention of multiple higher paths before observational truncation selects a set-like result.

8.5 Entanglement as nonfactorizable identity

Define entanglement by the failure

Equiv(A tensor B, A' tensor B') != Equiv(A,A') x Equiv(B,B').

The joint system possesses equivalences that cannot be decomposed into independent local equivalences.

8.6 Contextuality

Interpret contextuality as the impossibility of assigning globally compatible classical values across all local observational contexts.

8.7 Measurement as truncation

Model measurement as a map from a higher identity structure to a lower truncation level, such as

tau_0: R -> pi_0(R).

8.8 Decoherence as loss of accessible higher paths

Describe environmental decoherence as the suppression, dispersion, or practical inaccessibility of transformations connecting alternative presentations.

8.9 The Born rule problem

Identify the derivation of quantum probabilities from measures on equivalence spaces as a central unresolved requirement.

8.10 Quantum field theory

Outline how fields, particles, vacuum sectors, and scattering amplitudes might be represented within higher categories of local observables and processes.

9. Time, Causality, Irreversibility, and Entropy

9.1 Reversible equivalence and irreversible process

Distinguish invertible morphisms representing sameness from noninvertible morphisms representing genuine production, decay, measurement, or coarse-graining.

9.2 Time as directed composition

Propose that temporal order arises from the compositional sequence of noninvertible processes:

x -> y -> z.

9.3 The universe as an infinity-category

Model reality as an infinity-category in which higher morphisms are invertible but fundamental processes need not be.

9.4 Causal structure

Define causal accessibility through the existence of directed morphisms rather than through a background temporal coordinate.

9.5 Entropy as presentation multiplicity

Introduce an equivalence entropy:

$S_{eq}(m) = k_B \log \mathrm{Vol}(q^{-1}(m))$.

Interpret it as the multiplicity of distinguishable presentations realizing one coarse physical meaning.

9.6 Homotopy cardinality

Extend ordinary counting to include automorphism groups and higher topology so that highly symmetric presentations are weighted appropriately.

9.7 The second law

Reformulate thermodynamic evolution as movement toward macroscopic meanings with larger spaces of equivalent microscopic presentation.

9.8 Information loss and truncation

Relate entropy increase to the progressive forgetting of accessible distinctions among higher relational paths.

9.9 Memory and records

Define a physical record as a stable asymmetry in the equivalence network that constrains which past transformations remain reconstructible.

9.10 The arrow of time

Investigate whether temporal direction emerges from asymmetric coarse-graining, noninvertible composition, boundary conditions, or growth of equivalence-fiber volume.

10. The Genetic Code as a Finite Equivalence-First Toy Universe

10.1 Why use a nonphysical finite model

Explain that the genetic code is not proposed as evidence that fundamental physics is biological. It is used as a compact example of presentations, quotient meanings, adjacency, redundancy, and topology.

10.2 Sixty-four symbolic presentations

Treat the 64 codons as a complete finite presentation space generated by three positions and four symbols.

10.3 Twenty-one translated meanings

Treat twenty amino acids plus stop as quotient meanings under a many-to-one map from codons.

10.4 Synonymy as physical equivalence

Use synonymous codons as a finite analogy for distinct presentations representing the same output identity.

10.5 The codon cube as presentation geometry

Interpret single-coordinate changes as local movements through presentation space.

10.6 Self-loops as identity-preserving motion

After quotienting codons by translated identity, interpret synonymous adjacencies as transformations that alter presentation without altering meaning.

10.7 The G-A-C-U extremum

Describe how the G-A-C-U order and its reverse maximize synonymous nearest-neighbor connections among isotropic nucleotide orders.

10.8 An equivalence action for the cube

Define

$S_{eq}(\pi) = |E_\pi| - L(\pi)$,

where $|E_\pi|$ is the total edge count and $L(\pi)$ is the number of identity-preserving edges. The maximizing order for L minimizes this simple equivalence-loss action.

10.9 Rank deficiency and hidden degrees of freedom

Use the rank-10 structure of the 64 x 12 codon matrix as an example of visible coordinates exceeding independent degrees of freedom.

10.10 The pseudoinverse as latent reconstruction

Interpret the exact pseudoinverse as a canonical method for recovering lower-dimensional coordinate contributions from a higher-dimensional property field.

10.11 Limits of the analogy

Distinguish the demonstrated combinatorial facts of the genetic code from speculative claims about cosmology, simulation, design, or fundamental ontology.

10.12 The transferable principle

Extract the non-biological lesson: preferred geometry may be characterized by maximal local preservation of meaning under elementary transformation.

11. Mathematical Formalization and Computational Research Program

11.1 Selecting the foundational category

Compare infinity-groupoids, infinity-categories, higher toposes, dagger categories, monoidal categories, and derived geometric structures as candidate foundations.

11.2 Defining admissible physical equivalence

Specify criteria distinguishing true physical equivalence from arbitrary mathematical isomorphism or descriptive convenience.

11.3 Local observables and descent

Construct observables as functors or sheaves satisfying compatibility across equivalences and overlapping contexts.

11.4 Coherence functionals

Develop explicit measures of transport cost, loop obstruction, deformation complexity, and higher associativity failure.

11.5 Discrete equivalence networks

Build finite graph, simplicial-complex, cubical-complex, and higher-categorical models suitable for numerical study.

11.6 Continuum reconstruction

Study scaling limits in which discrete equivalence networks produce differentiable manifolds, connections, gauge fields, and effective metrics.

11.7 Homotopy-aware computation

Use persistent homology, higher-dimensional rewriting, proof assistants, type-theoretic computation, and categorical simulation to track equivalence structure.

11.8 Toy cosmologies

Construct evolving equivalence networks and test whether locality, dimensionality, causal cones, curvature, and stable defects emerge.

11.9 Quantum simulation

Develop finite path sums over groupoids and higher categories to compare with conventional Hilbert-space predictions.

11.10 Parameter estimation

Investigate whether coherence costs, automorphism volumes, or topological invariants can determine masses, couplings, or transition amplitudes.

11.11 Formal verification

Express core axioms in a homotopy-type-theoretic proof environment to separate rigorous consequences from interpretive conjecture.

11.12 Open datasets and reproducibility

Specify code, definitions, benchmark models, random controls, and numerical outputs required for independent replication.

12. Predictions, Falsifiability, Limitations, and Synthesis

12.1 Criteria for a physical theory

Require recovery of established low-energy physics, mathematical consistency, empirical distinguishability, calculational usefulness, and exposure to falsification.

12.2 Recovery of gauge theory

Show how gauge redundancy and holonomy should appear as special cases of local equivalence transport.

12.3 Recovery of general relativity

Identify the conditions under which coherence-derived geometry approximates Lorentzian spacetime and Einstein-like dynamics.

12.4 Recovery of quantum mechanics

Require a derivation or principled reconstruction of amplitudes, interference, tensor composition, measurement statistics, and the Born rule.

12.5 Novel predictions

Identify possible deviations involving topological sectors, coherence-dependent mass relations, restricted symmetry groups, emergent dimension, or nonstandard quantum correlations.

12.6 Distinguishing the framework from relabeling

Explain that merely rewriting known physics in categorical language is insufficient. The theory must derive structures, parameters, or constraints that existing formulations leave unexplained.

12.7 Mathematical risks

Discuss unresolved issues involving infinity-categorical foundations, measures on higher path spaces, continuum limits, and the definition of physically admissible equivalence.

12.8 Philosophical risks

Address the dangers of confusing descriptive equivalence with ontological identity and of treating mathematical elegance as empirical evidence.

12.9 The role of finite analogies

Clarify the legitimate and illegitimate uses of the genetic-code model and other finite quotient systems.

12.10 A staged research strategy

Propose progression from finite equivalence networks, to emergent metric models, to particle-like defects, to quantum amplitudes, and finally to gravitational and cosmological applications.

12.11 Unified interpretation

Synthesize matter, geometry, quantum coherence, entropy, and observation as different manifestations of equivalence structure and its obstruction.

12.12 Final conclusion

Conclude that a physical entity is not fundamentally a state occupying a location. It is a persistent, transportable, homotopy-coherent pattern within a category of processes. States are local presentations; particles are stable defects; charges are automorphism responses; geometry records the cost of identity transport; and classical facts are truncated remnants of higher equivalence. The foundational question of physics therefore changes from “What states exist?” to “Which transformations allow different presentations to count as the same reality?”

13. References

This section will contain the complete numbered bibliography. It will include foundational works in category theory, higher category theory, homotopy theory, homotopy type theory, univalent foundations, categorical quantum mechanics, gauge theory, general relativity, quantum field theory, topological phases, higher gauge theory, emergent spacetime, relational physics, information theory, thermodynamics, groupoid quantization, moduli spaces, and the genetic-code analysis motivating the finite toy model. Sections 1 through 12 will contain no inline references or hyperlinks.

1. From State-Based Physics to Equivalence-First Reality

1.1 The Conventional Ontology of States

Most formulations of physics begin by specifying a space of possible states. A classical particle is assigned a position and momentum. A field is assigned values throughout space and time. A quantum system is represented by a vector, ray, density operator, or algebraic state. Once the state space has been defined, dynamical laws determine how one state changes into another. In classical mechanics, the system traces a trajectory through phase space. In quantum mechanics, a state evolves through a Hilbert space. In field theory, a configuration changes according to differential equations derived from an action.

This framework has extraordinary predictive power, but it also carries an implicit ontology. It encourages the view that reality is fundamentally composed of states, while transformations, symmetries, coordinate changes, and equivalences are relations imposed upon those already existing states. A state is treated as the primary bearer of physical reality. Motion is then a sequence of states, symmetry is a relation among states, and measurement is a procedure that identifies or changes a state.

Yet the physical meaning of a state is rarely independent of context. A coordinate vector has meaning only relative to a coordinate system. A gauge potential is not uniquely physical because multiple potentials can represent the same electromagnetic field. A quantum state vector can be multiplied by a global phase without changing observable predictions. A spacetime event can be assigned different coordinates by different observers. Even the identity of a particle may depend on the symmetry structure used to classify it.

These examples suggest that the state itself may not be the final unit of physical reality. What appears as a single physical situation can possess many mathematical presentations, and what appears as a change of state can sometimes be only a change of description. The conventional ontology therefore contains a persistent tension: physics begins with states, but physical interpretation repeatedly requires identifying entire families of states as equivalent.

1.2 Equivalence as a Secondary Construction

In standard physical theories, equivalence usually enters after the state space has been introduced. One first defines a collection of states and then imposes a rule declaring that certain states represent the same physical situation.

In gauge theory, field configurations related by a gauge transformation are treated as physically equivalent. In general relativity, metric fields related by suitable coordinate transformations represent the same geometry. In quantum mechanics, vectors differing only by a nonzero complex scalar may represent the same ray. In statistical mechanics, many microscopic

configurations correspond to one macroscopic state. In particle physics, states are grouped according to symmetry representations, conserved charges, and transformation laws.

The usual procedure can be represented schematically as

presentation space -> equivalence relation -> physical state space.

Let P be a space of mathematical presentations. An equivalence relation ~ is imposed on P. The physical state space is then written as

M = P / ~.

The quotient M contains one representative for each equivalence class. Distinct elements p1 and p2 of P are treated as physically identical when

p1 ~ p2.

This construction is useful, but it places equivalence in a subordinate role. The presentations are defined first, and their physical redundancy is removed later. Equivalence appears as a correction to an overly large descriptive space.

The equivalence-first proposal reverses this order. It asks whether the relation among presentations is more fundamental than the presentations themselves. Perhaps a state does not possess identity before equivalence is specified. Perhaps the identity of a state is constituted by the transformations that preserve it, connect it to other presentations, and allow it to be recognized across changing contexts.

This possibility is already latent in modern physics. Gauge theories warn that mathematical variables may contain surplus structure. Relativity warns that coordinates do not possess invariant physical meaning. Quantum mechanics warns that observable identity may belong to a ray rather than to a vector. Topological phases warn that global relational structure can matter more than local numerical values. These theories do not merely tolerate equivalence. They depend upon it.

The central question is therefore whether equivalence should remain a secondary constraint on states or become the primary structure from which states are derived.

1.3 The Ontological Reversal

Univalent Relational Dynamics begins with an ontological reversal:

states are local presentations, while equivalences, transformations, and coherence relations are fundamental.

A presentation is a way in which an underlying physical structure appears within a chosen mathematical, experimental, or observational context. Different presentations may use different coordinates, gauges, bases, decompositions, or levels of description. They may look distinct while preserving the same physical identity.

Under this view, an isolated state is incomplete. Its meaning depends on the space of admissible transformations that connect it to other presentations. The object of interest is not merely a point x in a state space X. It is the network surrounding x:

- transformations from x to other presentations,
- transformations into x from other presentations,
- self-transformations of x,
- compositions among transformations,
- equivalences between alternative transformations,
- higher coherence relations among those equivalences.

The full physical structure is therefore not a set of states but a category, groupoid, higher category, or infinity-category of presentations and processes.

This shift changes the meaning of identity. In a state-based ontology, identity is assumed first and transformations act upon already identified objects. In an equivalence-first ontology, identity is reconstructed from the transformation structure. Two presentations count as the same physical entity because a suitable equivalence connects them. An entity persists because its equivalence structure can be transported coherently through change.

The reversal can be stated as follows:

A state is not an independently existing thing that later enters into relations. A state is a local appearance of a relational pattern that can be recognized through admissible transformations.

This position does not eliminate states from physics. States remain indispensable as local descriptions, boundary conditions, computational inputs, and experimental records. The claim is instead that states are not ontologically primitive. They are coordinate-like slices through a deeper structure of equivalence and process.

1.4 The Foundational Statement

The central proposition of this paper is:

Reality is not made of states; it is made of equivalence among states.

This statement must be interpreted carefully. It does not mean that all states are equivalent, that distinctions are unreal, or that physical differences can be ignored. Equivalence is always structured and conditional. Two presentations are physically equivalent only with respect to a specified class of admissible transformations, observables, and contexts.

The word equivalence therefore does not mean mere resemblance. It refers to a transformation that preserves the relevant physical structure. The preservation may be exact, approximate, local, effective, or scale-dependent. The theory must state what is preserved, which transformations are allowed, and how equivalences compose.

The foundational statement contains three claims.

First, physical identity is relational. A system is identified not by an inaccessible internal substance but by the pattern of transformations under which it remains coherent.

Second, physical distinction is also relational. Two presentations represent different realities when no admissible equivalence connects them, when they belong to disconnected sectors, or when the transformations between them alter physically invariant structure.

Third, the structure of equivalence is richer than a binary yes-or-no relation. There may be many distinct equivalences between the same presentations. Those equivalences may themselves be continuously deformed into one another. Some may form loops, carry phases, possess winding numbers, or exhibit higher obstructions. Physical identity is therefore not merely an equivalence class. It is a space of equivalences with its own internal geometry and topology.

The proposed ontology can be summarized schematically as

presentation + admissible transformations + coherence = physical identity.

The theory is therefore not about removing distinctions. It is about locating distinctions at the correct structural level.

1.5 Persistence Rather Than Substance

Classical metaphysics often asks what a thing is made of. Equivalence-first physics asks what allows a thing to remain identifiable through change.

A particle may move, accelerate, interact, change phase, or be described in another coordinate system while still being regarded as the same particle. A field may be expressed in multiple gauges while preserving the same observables. A geometric object may be parameterized in many ways while retaining the same intrinsic structure. A quantum system may undergo unitary change while preserving normalization and relational information.

These examples suggest that persistence is not passive. To persist is to maintain a coherent pattern across a family of transformations.

Let x(t) denote a sequence of presentations indexed by a parameter t. Conventional language treats x(t) as a changing state of one object. Equivalence-first language asks what justifies the phrase one object. The answer must be a family of transport maps

T(t1,t2): x(t1) -> x(t2)

that compose coherently:

T(t2,t3) o T(t1,t2) ~= T(t1,t3).

Identity through time is not contained in any individual x(t). It is contained in the coherent transport structure linking the presentations.

This perspective replaces substance with persistence criteria. An entity is not fundamental because it possesses an indivisible essence. It is physically real because a stable pattern of equivalence allows it to be followed, reconstructed, and distinguished through transformation.

The proposal also accommodates failure of persistence. A particle may decay when no transport map preserves its original identity type. A phase transition may occur when the equivalence structure reorganizes. A measurement may create a stable classical record by destroying or hiding higher paths between alternatives. A topological defect may persist precisely because no allowed local transformation can remove it.

Existence becomes a dynamical achievement:

To exist physically is to support a sufficiently stable and coherent pattern of identity-preserving transformations.

1.6 Scope of Univalent Relational Dynamics

Univalent Relational Dynamics is proposed here as a research program rather than a completed fundamental theory. Its purpose is to formulate an equivalence-first ontology in mathematical terms and determine whether it can reproduce, unify, or extend established physical structures.

The term univalent refers to the principle that structural equivalence can constitute identity. The term relational emphasizes that physical meaning belongs to transformations, compositions, and invariants rather than isolated points. The term dynamics indicates that equivalence is not static. Equivalence structures can be transported, created, broken, deformed, obstructed, and composed.

The program has several levels.

At the foundational level, it seeks precise definitions of presentation, physical equivalence, identity, observable, process, and coherence.

At the mathematical level, it employs category theory, higher category theory, groupoids, homotopy theory, homotopy type theory, moduli structures, sheaves, and related tools.

At the physical level, it aims to reinterpret geometry, gauge symmetry, particles, charge, mass, entanglement, measurement, entropy, and time as manifestations of equivalence structure.

At the computational level, it proposes finite relational networks, higher-dimensional complexes, categorical simulations, and homotopy-aware algorithms as test systems.

At the empirical level, it must eventually recover known physical predictions and produce additional constraints or observables that distinguish it from a purely philosophical reformulation.

The framework will therefore be judged by demanding standards. It must show that familiar state-based theories arise as effective or truncated descriptions. It must explain why particular equivalence structures are physically realized. It must derive meaningful dynamics rather than merely renaming gauge redundancy. It must clarify the origin of probabilities, dimensionality, causal order, and observable quantities. It must expose itself to mathematical failure and experimental refutation.

The genetic code will later be used only as a finite motivating model. Its many symbolic presentations, quotient meanings, identity-preserving adjacencies, redundant coordinates, and preferred topology offer a compact demonstration of the distinction between presentation and equivalence. The proposed physics does not depend on biological origin or biological interpretation.

The central objective is broader:

to construct a mathematical physics in which objects emerge from processes, identity emerges from equivalence, geometry emerges from transport, and persistence emerges from coherence.

The traditional question asks:

What states exist?

Univalent Relational Dynamics asks instead:

What transformations permit distinct presentations to belong to the same reality, and what physical structures arise when those transformations compose coherently or fail to do so?

2. Category Theory, Homotopy Type Theory, and Physical Identity

2.1 Categories of Physical Processes

Category theory begins not with the internal composition of isolated objects, but with objects together with the transformations connecting them. A category C consists of objects x, y, z and morphisms

f: x -> y,

g: y -> z,

together with a rule of composition

g o f: x -> z.

Every object x also possesses an identity morphism

id_x: x -> x,

and composition satisfies associativity:

h o (g o f) = (h o g) o f.

These axioms appear elementary, but they contain a decisive shift in emphasis. An object is understood through the processes in which it can participate, the transformations it admits, and the ways those transformations compose. The category does not require that an object be reduced to an underlying substance or set of elementary constituents. Its identity is expressed relationally.

In a physical interpretation, objects may represent experimental contexts, field configurations, boundary conditions, local coordinate descriptions, preparation procedures, or temporary presentations of a physical system. Morphisms may represent evolutions, interactions, measurements, coordinate transformations, coarse-grainings, embeddings, or transfers of information.

The meaning of a physical presentation is then inseparable from the morphisms entering and leaving it. A state that cannot be prepared, transformed, observed, or related to another state has no operational physical content. This suggests that the categorical description may be more fundamental than the conventional state-space description.

The ordinary state-space picture can still be recovered. A set of states can be treated as a category in which the only morphisms are identity morphisms. But such a category is relationally impoverished. It records which states exist while forgetting the processes connecting them. Physical theories are rarely so sparse. They contain transformations,

symmetries, histories, interactions, and conservation laws. Category theory therefore provides a natural language for treating process as fundamental.

In Univalent Relational Dynamics, an apparent state is an object only provisionally. Its full physical meaning resides in the surrounding category of transformations. The ontology is shifted from isolated states to composable processes.

2.2 From Categories to Higher Categories

Ordinary category theory treats morphisms as primitive arrows. However, physics often requires relations among transformations themselves.

Two different paths may connect the same initial and final presentations. Two gauge transformations may differ by a continuous deformation. Two histories may be related by a symmetry. Two computational procedures may implement the same effective physical process. Two coordinate transformations may produce equivalent descriptions of the same geometric relation.

These situations require morphisms between morphisms.

Suppose

f, g: x -> y.

A 2-morphism

alpha: f => g

represents a transformation or equivalence between the processes f and g. In a 2-category, objects are connected by 1-morphisms, and 1-morphisms are connected by 2-morphisms. The 2-morphisms may be composed vertically and horizontally, subject to coherence conditions.

The hierarchy need not stop there. Two 2-morphisms may themselves be related by a 3-morphism. Two 3-morphisms may be related by a 4-morphism. Continuing this construction yields higher categories and, in the limiting case, infinity-categories.

This hierarchy is not merely decorative mathematical structure. It expresses increasingly refined forms of physical identity.

At the first level, one asks whether two presentations are connected.

At the second level, one asks whether two transformations between those presentations are themselves equivalent.

At the third level, one asks whether two equivalences between transformations are coherently related.

Higher levels record the consistency of entire systems of transformations.

Physical theories already contain shadows of this hierarchy. Gauge transformations act on fields. Transformations between gauge transformations appear in higher gauge theories. Homotopies relate paths. Homotopies between homotopies relate surfaces. Extended objects such as strings, membranes, and defects naturally require transformations associated with regions of dimension greater than one.

A state-based ontology tends to compress these structures into auxiliary mathematical machinery. An equivalence-first ontology treats them as part of physical identity itself.

The identity of a physical object is therefore not exhausted by the statement that it remains the same under a single transformation. It includes the entire coherent hierarchy of ways in which its transformations can be compared and composed.

2.3 Infinity-Groupoids and Reversible Structure

A groupoid is a category in which every morphism is invertible. For every arrow

f: x -> y,

there exists an inverse

f^-1: y -> x

such that

f^-1 o f = id_x

and

f o f^-1 = id_y.

A groupoid therefore describes a collection of objects together with explicit equivalences among them. Unlike a quotient set, it does not merely declare that x and y belong to the same equivalence class. It records the particular transformations connecting them.

An infinity-groupoid extends this idea to all higher levels. Objects are connected by invertible 1-morphisms, 1-morphisms are connected by invertible 2-morphisms, and this hierarchy continues indefinitely.

Infinity-groupoids provide a natural mathematical model for spaces. Points correspond to objects. Paths correspond to 1-morphisms. Deformations between paths correspond to 2-morphisms. Deformations between those deformations correspond to higher morphisms.

The physical importance of this structure is that identity becomes spatially and topologically rich. There may be several distinct paths by which one presentation is identified with another. Some paths may be deformable into each other, while others may belong to different topological classes. Closed paths may form nontrivial loops. Higher-dimensional families of paths may contain obstructions or winding structures.

Consider a presentation x. The self-equivalences

x -> x

form the automorphism structure of x. A nontrivial loop beginning and ending at x may encode a symmetry, phase, holonomy, or conserved transformation. The physical identity of x therefore includes not only its position in a presentation space but also the topology of the paths returning to it.

This gives a richer account of sameness. Two presentations can be equivalent in more than one way, and the differences among those ways may be physically measurable.

In a gauge theory, distinct gauge transformations connecting the same field descriptions may differ by topologically nontrivial structure. In quantum theory, alternative histories may accumulate different phases. In gravitational physics, parallel transport along different paths may produce different orientations. These phenomena indicate that the space of equivalences matters, not merely the existence of equivalence.

Univalent Relational Dynamics therefore uses infinity-groupoid structure to model the reversible aspect of reality. Reversible transformations express ways in which presentation may change while physical identity is retained.

Irreversible processes will later require a broader infinity-category in which some primary morphisms do not possess inverses. The infinity-groupoid remains essential because it describes the internal architecture of physical sameness.

2.4 Identity as Path

Homotopy type theory interprets identity as a structured type rather than a simple logical predicate.

For elements x and y of a type A, the identity type

Id_A(x,y)

contains witnesses that x and y are identical. Instead of treating equality as an external statement, the theory treats an identification as an object that can itself be studied.

In geometric language, an inhabitant

p: Id_A(x,y)

can be interpreted as a path from x to y.

This path interpretation transforms identity from a binary assertion into a space of possible identifications. There may be no path between x and y, one path, or many inequivalent paths. Paths may compose:

p: x = y,

q: y = z,

q o p: x = z.

Paths may be reversed:

p^-1: y = x.

Every point has a constant path:

refl_x: x = x.

These operations resemble the structure of a groupoid. Higher identity types then produce paths between paths:

alpha: p = q.

The hierarchy continues with identities between higher identities.

Applied physically, this means that the statement “x and y represent the same physical entity” is incomplete unless one specifies how they are identified. The identification may contain operational, geometric, topological, or dynamical information.

For example, two coordinate descriptions of a spacetime region may be identified by a specific coordinate transformation. Two gauge potentials may be identified by a specific gauge function. Two experimental preparations may be identified by a calibrated transformation between apparatus settings. Two time-separated presentations of a particle may be identified through a worldline and a transport law.

The path is not merely evidence of sameness. It is the structure that makes sameness meaningful.

This viewpoint also clarifies why identity can be nontrivial even when the initial and final presentations coincide. A path

p: x = x

need not be equivalent to the constant path refl_x. Such a loop may represent a symmetry operation, phase rotation, winding, or holonomy.

Physical identity therefore has internal topology. Returning to the same presentation does not imply that nothing occurred. A loop may preserve the endpoint while altering phase, orientation, internal state, or higher coherence.

The path interpretation replaces static identity with transportable identity. A physical entity persists not because an unchanging substance remains hidden beneath appearances, but because paths of identification connect its changing presentations.

2.5 Univalence as a Physical Principle

The univalence axiom states, in mathematical form, that equivalent structures may be identified. For types A and B, it establishes a correspondence between identity of types and equivalence of types:

Id(A,B) ~= Equiv(A,B).

This principle does not assert that every resemblance is an identity. It applies to genuine structural equivalence: a reversible correspondence preserving the relevant organization.

Transposed into physics, the principle becomes

Id_phys(x,y) ~= Equiv_phys(x,y).

Two presentations count as the same physical reality when an admissible physical equivalence connects them. Conversely, an identification of physical presentations should correspond to an equivalence preserving the structures that define their physical meaning.

The adjective admissible is essential. Not every mathematical bijection, isomorphism, or formal transformation is physically meaningful. A physical theory must specify which structures are preserved.

An admissible equivalence might preserve some combination of:

observable probabilities,

causal relations,

interaction structure,

symmetry representation,

conserved quantities,

topological class,

composition laws,

boundary conditions,

or experimentally accessible predictions.

The physical form of univalence therefore depends on the theory's chosen notion of structure.

This principle sharpens ideas already present in modern physics. Coordinate-related descriptions in general relativity are not merely numerically similar; under appropriate conditions they represent one spacetime geometry. Gauge-related field configurations represent one physical situation. Different bases of a Hilbert space represent the same abstract quantum structure. Unitary equivalence can preserve the operational content of a quantum theory.

The proposed framework generalizes this pattern. It treats physical identity as generated by admissible equivalence rather than assigned independently.

This has an important consequence. If two presentations are equivalent, transporting a physical statement from one to the other should preserve its truth and operational content. Equivalence permits substitution.

If

e: x ~= y,

then any physically meaningful construction F should provide a corresponding transport

F(e): F(x) ~= F(y).

Observables, interactions, and derived structures must respect this transport. Otherwise they depend on presentation rather than physical reality.

Univalence thereby becomes both an ontological and methodological principle. Ontologically, equivalent presentations constitute one physical identity. Methodologically, valid physical laws must be invariant under transport across admissible equivalence.

2.6 Equivalence Spaces Rather Than Equivalence Predicates

An ordinary equivalence relation answers a yes-or-no question:

Is x equivalent to y?

This compresses all possible identifications into a single Boolean result. If x ~ y, the relation does not record whether there is one transformation between them or many, whether those

transformations are continuously connected, whether they form distinct topological sectors, or whether some transformations possess nontrivial automorphisms.

An equivalence-first physics must preserve this information.

For presentations x and y, define an equivalence space

Equiv_phys(x,y).

The elements of this space are admissible physical equivalences from x to y. The space may itself possess topology, geometry, probability, cost, phase, or higher categorical structure.

If Equiv_phys(x,y) is empty, the presentations are physically disconnected under the allowed transformations.

If it contains one contractible family of equivalences, the identification is essentially unique.

If it contains several disconnected components, there are qualitatively distinct ways to identify x and y.

If it contains nontrivial loops, the equivalences possess their own symmetries.

If it contains higher homotopy, there are nontrivial coherence structures among transformations.

This enrichment may carry direct physical meaning. Distinct components could correspond to topological sectors. Loops could correspond to phases or internal symmetries. The geometry of the equivalence space could determine action cost. Its measure could influence quantum amplitudes. Its automorphism structure could affect statistical weighting.

The physical state space should therefore not be represented merely as a set of equivalence classes. It should be represented as a moduli groupoid, homotopy quotient, higher stack, or related structure that retains the internal organization of equivalence.

This distinction can be illustrated schematically.

The quotient-set description is

M_set = P / ~.

The equivalence-space description is

M_higher = P // E.

The first remembers only which presentations are grouped together. The second remembers the transformations, loops, stabilizers, and higher relations within each grouping.

Two theories may have the same quotient set while differing in their equivalence spaces. They would agree on the number of physical classes but disagree on symmetry, topology, phase structure, and dynamics.

For Univalent Relational Dynamics, such theories are physically distinct.

2.7 Identity Without Primitive Objects

The strongest form of the equivalence-first proposal questions whether objects must be assumed at all.

In ordinary language, relations appear to require preexisting things that stand in those relations. Category theory suggests a subtler possibility. An object can often be characterized by the total pattern of morphisms connecting it to other objects.

A classical categorical principle states that objects are determined, up to isomorphism, by their relationships to all other objects. Rather than opening an object and inspecting an intrinsic substance, one studies the morphisms into and out of it.

For an object x in a category C, consider the assignment

y -> Hom_C(y,x).

This records all morphisms from each test object y into x. Under suitable conditions, the complete pattern determines x up to canonical equivalence.

The physical analogue is that an entity may be determined by all possible preparations, interactions, transformations, and observations involving it. Its identity is exhausted by its relational profile.

A particle, for example, may be characterized through:

how it transforms under symmetries,

which interactions it permits,

which conserved quantities accompany it,

how it composes with other particles,

which scattering amplitudes involve it,

which topological sector contains it,

and which automorphisms preserve it.

No additional hidden substance need be postulated beyond this coherent network.

This view does not reduce reality to arbitrary external observation. The relational structure is not merely a list of human measurements. It is the objective compositional architecture of possible physical processes.

An object emerges as a stable node-pattern in that architecture. It is a center of relational consistency rather than a primitive bearer of properties.

This suggests a sequence of reconstruction:

processes determine compositional structure,

compositional structure determines equivalence,

equivalence determines identity,

and stable identity patterns appear as objects.

The traditional ontology begins with objects and derives relations. The equivalence-first ontology begins with composable transformation and derives objects as invariants of that transformation structure.

The strongest statement of the proposal is therefore:

Physical objects are not the foundations of relations. Physical objects are persistent solutions to relational coherence.

Category theory provides the language of composition. Higher category theory provides relations among processes. Homotopy type theory provides identity as path. Univalence identifies structural equivalence with sameness. Together, these form the mathematical foundation for a physics in which identity is not assumed but constructed.

3. The Axioms of Univalent Relational Dynamics

3.1 The Presentation Axiom

The first axiom states that every observed or mathematically represented state is a presentation relative to a context. A presentation is not assumed to be the final physical object. It is the form in which some underlying relational structure appears within a selected coordinate system, gauge, basis, scale, apparatus, language, or experimental arrangement.

Let P denote a space of presentations. An element

p in P

may be a coordinate description, field configuration, quantum state vector, experimental record, graph, tensor, boundary condition, or local model. The same physical structure may admit many such presentations.

The presentation axiom can be stated as follows:

Every state accessible to theory or experiment is context-dependent, and no single presentation is presumed to exhaust physical identity.

This axiom does not imply that physical reality is subjective. The context determines how reality is represented, not whether reality exists. Different observers may employ different coordinates while describing the same spacetime geometry. Different gauges may represent the same field. Different bases may represent the same linear operator. The variations belong to the presentation; the invariant relational structure belongs to the physical identity.

A presentation therefore has two aspects. It contains genuine physical information, but it may also contain representational redundancy. The purpose of the theory is to distinguish what changes when the presentation changes from what remains invariant under admissible transformation.

Suppose p and q are two presentations. Their apparent numerical or symbolic difference does not establish that they correspond to different physical realities. One must ask whether there exists an admissible equivalence

e: p ~= q.

Conversely, two presentations that resemble each other closely may still represent different physical structures if no admissible equivalence preserves the relevant observables, causal relations, boundary conditions, or composition laws.

The presentation axiom therefore places physical interpretation before literal comparison of state descriptions. It warns against identifying coordinates with geometry, gauge potentials with observables, vectors with rays, or microscopic labels with macroscopic identity.

This axiom also introduces scale dependence. A presentation may be sufficient at one scale and inadequate at another. A fluid element may be treated as a point in hydrodynamics, a molecular ensemble in statistical mechanics, or a quantum system at shorter scales. These presentations are not simply different levels of detail. They are connected by maps that preserve some structures while discarding others.

The physical problem is therefore not merely to enumerate states. It is to identify the transformations connecting presentations and determine which aspects survive those transformations.

3.2 The Univalent Identity Axiom

The second axiom places admissible equivalence at the center of physical identity.

For presentations x and y, define

Id_phys(x,y)

as the space of physical identifications between them, and define

Equiv_phys(x,y)

as the space of admissible structure-preserving equivalences.

The univalent identity axiom states

Id_phys(x,y) ~= Equiv_phys(x,y).

This means that physical sameness is constituted by admissible equivalence. Two presentations are not first declared identical and then discovered to be equivalent. Their equivalence supplies the content of their identity.

The axiom must not be interpreted as saying that every mathematical equivalence is physically meaningful. The category of admissible transformations must be specified by the theory. A transformation qualifies as a physical equivalence only when it preserves the structures that define the physical situation.

Depending on the theory, those structures may include:

observable probabilities,

causal order,

interaction rules,

symmetry representations,

conserved quantities,

topological class,

boundary data,

metric relationships,

composition laws,

or experimentally reproducible predictions.

The admissibility criterion is therefore part of the physical law, not merely a mathematical convenience.

The axiom also implies transport of truth. If x and y are physically equivalent and a proposition or observable is well defined on x, it must be transportable to y.

Given

e: x ~= y,

a physical construction F must induce

F(e): F(x) ~= F(y).

A quantity that changes only because one has replaced x with an equivalent presentation is not a physical observable. It is presentation-dependent data.

This leads to a sharp distinction between descriptive variation and physical change.

A descriptive variation occurs when

x ~= y.

A physical change occurs when the transformation alters invariant structure or moves the system into a different physical equivalence class.

The univalent identity axiom therefore gives a criterion for deciding when a transformation represents motion, interaction, or evolution and when it represents only a change of description.

It also allows identity to be nonunique. There may be several admissible equivalences connecting x and y. These alternatives may differ by topology, phase, winding, orientation, or higher coherence. Physical sameness is therefore not necessarily represented by one canonical map. It may form an entire space of identifications.

The multiplicity of equivalences can itself become observable. Interference, holonomy, geometric phase, and topological sector structure may all arise from distinct ways of identifying presentations.

3.3 The Higher-Coherence Axiom

The third axiom states that equivalences between equivalences possess physical significance whenever they affect composition, transport, interference, or global consistency.

Suppose two transformations connect the same presentations:

f, g: x -> y.

An ordinary state-based theory may compare only their endpoints. An equivalence-first theory must also ask whether f and g are themselves related by a higher transformation:

alpha: f => g.

If such a transformation exists, f and g may represent two presentations of the same process. If no such transformation exists, they may belong to different process sectors even though they share endpoints.

The hierarchy continues. Two 2-morphisms

alpha, beta: f => g

may themselves be related by a 3-morphism. Higher coherence records whether alternative identifications among processes are compatible.

The higher-coherence axiom states:

Physical equivalence is complete only when the composition of equivalences is itself coherently specified at all relevant levels.

This requirement prevents ambiguity from accumulating when local transformations are combined. If a system is transported through several contexts, the final identification should not depend arbitrarily on how the composition is parenthesized, decomposed, or represented.

For three composable transformations,

f: x -> y,

g: y -> z,

h: z -> w,

one compares

h o (g o f)

with

(h o g) o f.

In an ordinary category, these are exactly equal. In a weak higher category, they may be connected by a specified associator. Further coherence laws ensure that different sequences of associators lead to compatible outcomes.

The physical interpretation is that multi-stage processes require consistency beyond pairwise equivalence. Local agreements must fit together into global structure.

Failure of higher coherence can signal a genuine physical obstruction. Such failure may appear as anomaly, curvature, topological defect, frustration, or incompatibility among local descriptions.

A field may be locally trivializable but globally nontrivial. Local coordinate patches may each be well behaved while failing to glue into a global chart. Local gauge choices may exist while transition functions around loops retain nontrivial holonomy. These are not defects of notation. They are physical or geometric consequences of incomplete global coherence.

The axiom therefore elevates coherence conditions from technical bookkeeping to possible carriers of physical law.

3.4 The Observable-Descent Axiom

The fourth axiom concerns measurable quantities.

An observable is physically meaningful only if it is compatible with admissible equivalence. If x and y are equivalent presentations, the observable must assign them the same value or appropriately equivalent values.

In its simplest form,

x ~= y implies O(x) = O(y).

In a richer setting, the observable may take values in another category, so strict equality is replaced by equivalence:

x ~= y implies O(x) ~= O(y).

This requirement is called descent because the observable must pass from the presentation space to the physical moduli structure. It must not depend on arbitrary representational choices.

Let

q: P -> M

map presentations to physical meanings. A presentation-dependent quantity is a map

O_P: P -> V.

For O_P to define a physical observable on M, there must exist a map

O_M: M -> V

such that

O_P = O_M o q.

This means that O_P is constant, or coherently constant, on each equivalence fiber.

The observable-descent axiom states:

Every physical observable must factor through the equivalence structure.

Gauge-invariant quantities satisfy this condition. Coordinate-independent scalars satisfy it. Probabilities associated with quantum rays rather than vector phases satisfy it. Macroscopic observables in statistical mechanics descend from large sets of microscopic presentations.

The axiom also clarifies why some mathematically definable quantities are not directly physical. A coordinate component, gauge-dependent phase, or basis-dependent matrix entry may be useful for calculation without corresponding to an invariant observable.

This does not make presentation-dependent quantities meaningless. They can function as local coordinates, auxiliary fields, computational devices, or intermediate variables. Their physical interpretation, however, arises only through combinations that descend appropriately.

In higher-categorical settings, descent may require more than equality on equivalent objects. The observable must also respect equivalences between transformations and the coherence among those equivalences. A local observable that agrees pairwise but fails under triple overlap may not define a global physical quantity.

The observable-descent axiom therefore links measurement to consistency across the entire relational structure.

3.5 The Compositionality Axiom

The fifth axiom states that physical processes compose.

If

f: x -> y

is a process from x to y, and

g: y -> z

is a process from y to z, then there exists a composite process

g o f: x -> z.

This simple rule is fundamental because it permits complex physical histories to be constructed from simpler processes.

The compositionality axiom states:

Physical processes can be joined whenever the output boundary of one process matches the input boundary of another, and the resulting composition is associative up to the appropriate level of coherence.

Composition supplies the architecture of causation, experiment, computation, interaction, and temporal sequence. A preparation is followed by an evolution, which is followed by a measurement. A particle enters an interaction region and later exits. A local coordinate transformation is followed by another transformation. A signal passes through successive channels.

The identity morphism

id_x: x -> x

represents the neutral process that leaves x unchanged. The existence of identity processes is essential for comparing transformations and defining symmetry.

Composition also separates reversible equivalence from irreversible process. An equivalence e has an inverse e^-1. A decay, measurement, coarse-graining, or dissipative transformation may not.

Univalent Relational Dynamics therefore distinguishes two classes of morphism:

invertible morphisms, representing changes of presentation that preserve identity,

and noninvertible morphisms, representing genuine physical production, loss, selection, or irreversible transformation.

The complete physical structure is not necessarily an infinity-groupoid in which all morphisms are invertible. It is more naturally an infinity-category whose higher morphisms may be invertible while some basic processes are directional.

Compositionality also enables locality. A global process may be decomposed into local subprocesses. Conversely, a collection of local subprocesses may define a global transformation when their shared boundaries match coherently.

This provides a route from microscopic relational rules to macroscopic dynamics. Rather than postulating a global trajectory, one builds a network of composable processes and studies the structures that emerge from repeated composition.

3.6 The Local-to-Global Axiom

The sixth axiom states that global physical structures arise by coherently combining compatible local data.

Physics is often formulated locally. Fields are defined in neighborhoods. Coordinate charts cover manifolds. Gauge potentials are chosen on patches. Measurements are performed within finite regions. Observers access only limited contexts.

Yet physical theories must explain how these local descriptions combine into a global object.

Let {U_i} be a collection of local contexts. Suppose each U_i carries a presentation x_i. On overlaps U_i intersect U_j, there are transition equivalences

e_ij: x_i ~= x_j.

For the local presentations to define one global physical structure, the transition maps must satisfy compatibility conditions. On triple overlaps, one requires

e_jk o e_ij ~= e_ik.

In higher settings, this compatibility is witnessed by specified higher morphisms, and those witnesses must satisfy further coherence conditions on quadruple overlaps and beyond.

The local-to-global axiom states:

Compatible local presentations and equivalences determine a global physical structure when their overlap data satisfy the required hierarchy of coherence.

This axiom is essential for emergent spacetime, gauge theory, quantum field theory, and distributed relational models.

It also gives physical meaning to obstruction. Local data may be individually consistent while failing to glue globally. The failure can indicate nontrivial topology, curvature, anomaly, defect, or conserved charge.

A magnetic monopole, for example, may be represented through gauge potentials that exist locally but cannot be combined into one globally regular potential. The global obstruction carries physical information.

Similarly, a topological phase may possess no distinctive local order parameter while differing globally through its pattern of transition functions and loops.

The local-to-global axiom therefore rejects the assumption that global reality is merely the sum of local states. Global identity depends on how local presentations are related.

The principle may be summarized as

local objects + overlap equivalences + higher coherence = global object.

In an equivalence-first theory, the gluing data may be more fundamental than the local objects being glued.

3.7 The Dynamical-Coherence Axiom

The seventh axiom introduces change.

Equivalence structures are not static. They may be created, transported, strengthened, weakened, broken, merged, split, or obstructed. A physical theory must therefore describe the dynamics of coherence itself.

The dynamical-coherence axiom states:

Physical evolution is governed by laws controlling the formation, transport, deformation, stabilization, and failure of equivalence.

A conventional theory assigns an action to a field or trajectory. Univalent Relational Dynamics seeks an action or rule assigning cost to relational transformations and coherence failures.

Let T_ij denote a transport map between neighboring presentations x_i and x_j. Let C(T_ij) measure the cost of maintaining equivalence across that connection. Let K(T_loop,id) measure the failure of transport around a loop to return to the identity.

A schematic action may take the form

S = sum_(i,j) C(T_ij)

- lambda sum_loops K(T_loop,id)
- mu sum_i B(x_i).

The first term measures local transport cost. The second penalizes or weights failure of loop coherence. The third represents local boundary, compatibility, or maintenance cost.

The dynamics may then be determined by extremizing S, summing over equivalence structures, or applying local update rules.

This leads to a possible foundational principle:

Realized physical structures maximize coherent equivalence subject to the cost of maintaining that coherence.

The principle does not require that all presentations become equivalent. A completely undifferentiated structure would contain no stable distinctions. Physical organization may instead arise from a balance between coherence and obstruction.

Too little equivalence yields disconnected fragments with no persistent identity. Too much equivalence collapses all distinctions. Complex reality may occupy an intermediate regime in which some transformations preserve identity while others generate genuine change.

Curvature, mass, charge, and topological stability may eventually be interpreted as different forms of coherence cost or obstruction.

Dynamics then becomes the evolution not merely of states but of the rules determining which states count as the same.

3.8 The Finite-Observation Axiom

The eighth axiom addresses the relationship between the full higher structure and actual observation.

No finite experiment accesses an unlimited hierarchy of equivalences, paths between paths, and higher coherence data. Measurement reveals only a truncated portion of the relational structure.

The finite-observation axiom states:

Every actual observation accesses a finite or effectively truncated image of the underlying higher equivalence structure.

Let R denote a higher relational object. A classical observation may apply a truncation map

tau_0: R -> pi_0(R),

which retains disconnected outcome sectors while forgetting paths and higher paths connecting presentations within or between those sectors.

A less severe observation may preserve some groupoid structure:

tau_1: R -> R_1,

retaining objects and ordinary equivalences while discarding higher relations.

Different measurement regimes may therefore correspond to different truncation levels.

The axiom offers a possible interpretation of classicality. Classical outcomes appear set-like because higher identity structure has become inaccessible, suppressed, averaged over, or recorded only through a coarse quotient.

Quantum coherence, by contrast, requires continued accessibility of multiple paths and their relative phases. Decoherence occurs when environmental interaction disperses the information needed to compare those paths.

Measurement can then be represented not as the mysterious destruction of reality, but as a reduction in the level of relational structure available to the observer.

This does not by itself solve the measurement problem. It does, however, reformulate it.

The question is no longer only:

Why does a quantum state collapse to one classical state?

It becomes:

Why does an observer obtain access only to a truncated relational object in which higher equivalences no longer contribute to observable interference?

The finite-observation axiom also prevents the theory from identifying physical reality with any one measurement record. A record is a presentation produced by a particular truncation process. It may be stable, objective, and reproducible without exhausting the higher structure from which it emerged.

Together, the eight axioms define the conceptual core of Univalent Relational Dynamics.

Presentations replace primitive states.

Admissible equivalence constitutes identity.

Higher coherence governs consistency among transformations.

Observables descend through equivalence.

Processes compose.

Local structures glue through coherent transition data.

Dynamics governs the evolution of equivalence itself.

Observation reveals truncated images of the full relational hierarchy.

The resulting theory does not begin by asking which isolated states are real. It begins by asking which presentations can be coherently identified, how those identifications compose, where they fail, and what physical structures emerge from that pattern of persistence and obstruction.

4. Presentation Spaces, Moduli Structures, and Homotopy Quotients

4.1 The Space of Presentations

A physical theory rarely begins directly with invariant reality. It begins with descriptions. These may be coordinate assignments, field components, wavefunctions, basis vectors, gauge potentials, graphs, tensor networks, boundary data, detector records, or numerical configurations. Each such object is a presentation of physical structure rather than necessarily the physical structure itself.

Let

P

denote a space of presentations. An element

p in P

is a mathematically explicit description available to calculation or observation.

The term space is used broadly. P may be a set, topological space, manifold, Hilbert space, category, simplicial complex, function space, or higher geometric object. Its exact form depends on the theory being studied.

A presentation space usually contains more information than is physically distinguishable. For example, a vector psi and the vector

exp(i theta) psi

may represent the same quantum ray. Two gauge potentials may represent the same electromagnetic field. Two coordinate expressions may describe the same spacetime geometry. Two microscopic configurations may correspond to the same thermodynamic macrostate.

This redundancy is not an accidental flaw. It frequently makes local calculation possible. Coordinates, gauges, bases, and parameterizations provide manageable representations of structures that may be difficult to describe globally.

The problem is that physical ontology cannot simply be identified with P. If it were, a change of coordinates would become a change of universe, a gauge transformation would create a new electromagnetic reality, and multiplication of a quantum vector by a global phase would produce a new physical state.

A presentation must therefore be accompanied by information specifying which changes alter physical content and which merely alter representation.

For every p in P, one must ask:

Which other presentations represent the same physical structure?

Which transformations connect them?

Which observables remain invariant under those transformations?

Which transformations are reversible?

Which transformations preserve only part of the structure?

Which presentations cannot be connected at all?

The answers define the relational architecture of the theory.

In conventional formulations, P is often treated as primary and redundancy is removed afterward. In Univalent Relational Dynamics, P is regarded as a local descriptive layer. Physical reality is reconstructed from P together with the full system of admissible transformations acting within it.

The appropriate starting object is therefore not P alone but something like

(P,E),

where E records equivalences and processes among presentations.

This pair contains more physical information than either component separately. P without E cannot distinguish physical difference from descriptive difference. E without P has no explicit local forms upon which transformations act.

The presentation space is therefore necessary but not sufficient. It is the visible arena in which a deeper equivalence structure becomes mathematically expressible.

4.2 Equivalence Relations and Groupoid Structure

The simplest method for removing redundancy is to introduce an equivalence relation

~

on P.

For p,q in P,

p ~ q

means that p and q represent the same physical situation.

An equivalence relation satisfies three properties:

p ~ p,

p ~ q implies q ~ p,

and

p ~ q and q ~ r imply p ~ r.

These are reflexivity, symmetry, and transitivity.

This construction partitions P into equivalence classes. Each class contains all presentations declared physically identical.

Although useful, the equivalence relation records only whether two presentations are equivalent. It does not record how they are equivalent.

Suppose two coordinate systems describe the same geometry. The statement

p ~ q

forgets the coordinate transformation between them.

Suppose two gauge potentials describe the same gauge field. The equivalence relation forgets the gauge function connecting them.

Suppose two paths between the same endpoints are homotopic. The relation forgets the deformation turning one path into the other.

To retain this information, the equivalence relation must be enriched into a groupoid.

A groupoid G consists of:

objects, representing presentations;

morphisms, representing explicit equivalences;

identity morphisms;

composition of compatible morphisms;

and inverses for all morphisms.

If

e: p -> q

is an equivalence, then there exists

e^-1: q -> p,

with

$e^{-1} \circ e = id_p$

and

$e \circ e^{-1} = id_q$.

The groupoid remembers not merely that p and q belong to the same equivalence class, but the distinct ways they can be identified.

There may be several morphisms

$e1, e2, \ldots : p \to q$.

These may correspond to different transformations, paths, phases, windings, coordinate maps, or symmetry operations.

The self-equivalences

$a : p \to p$

form the automorphism group

$Aut(p)$.

This group may contain important physical information. It can encode internal symmetry, stabilizers, residual gauge transformations, geometric isotropy, or degeneracy.

A quotient set treats all objects in an equivalence class as one point. A groupoid retains the internal traffic among them.

The difference is analogous to replacing a transportation map with a list of cities. The list records which destinations exist, but the map records how they are connected, which routes form loops, which routes are interchangeable, and which cities possess unusual symmetry.

For equivalence-first physics, the map is closer to the ontology than the list.

The physical content is not exhausted by the collection of equivalence classes. It includes the morphisms within each class and the composition rules among them.

4.3 The Moduli Space of Physical Meanings

Let

$q : P \to M$

be a map from the presentation space P to a space M of physical meanings.

For p in P, the value

q(p)

is the physical identity represented by p.

Two presentations p and q represent the same meaning when

q(p) = q(q).

Equivalently,

p ~ q.

The space M is often called a moduli space. Each point of M represents one equivalence class of physically identical presentations.

The word moduli emphasizes that the elements of M classify physically distinct structures after redundant descriptions have been identified.

Examples of moduli-like constructions occur throughout mathematical physics. One may consider spaces of geometries modulo coordinate transformations, gauge fields modulo gauge transformations, solutions of differential equations modulo symmetry, or quantum states modulo global phase.

The map

q: P -> M

separates descriptive variation from physical variation.

Movement within a fiber of q changes the presentation while preserving the physical meaning.

Movement between fibers changes the physical meaning.

For m in M, the fiber

q^-1(m)

is the collection of all presentations representing m.

This gives a useful decomposition:

P = union over m in M of q^-1(m).

Each fiber is an equivalence family. The moduli space records which family a presentation belongs to.

However, the moduli-space picture immediately raises questions.

Are all fibers structurally identical?

Do some meanings admit more presentations than others?

Do some presentations possess larger automorphism groups?

Are the fibers connected?

Do they contain nontrivial loops?

Can one choose a preferred presentation continuously across M?

Do singular points occur where the symmetry increases?

These questions cannot be answered from the bare set of points in M. They require the groupoid or higher structure that was compressed when the quotient was formed.

Even so, the moduli-space concept is essential because it formalizes the distinction between presentation and physical identity.

P is the descriptive space.

M is the space of meanings.

q is the interpretation map.

The central problem of Univalent Relational Dynamics is to determine whether M should be treated as fundamental, or whether even M is only a low-resolution shadow of a richer homotopy quotient retaining the full structure of equivalence.

4.4 Why an Ordinary Quotient Is Insufficient

The ordinary quotient

M_set = P / ~

replaces each equivalence class with a single point.

This operation removes descriptive redundancy, but it can remove physically significant information as well.

Suppose p has a nontrivial symmetry group Aut(p). In the quotient set, the point [p] does not record whether p has no nontrivial symmetry, a finite symmetry group, or a continuous symmetry group.

Suppose two presentations p and q are connected by several inequivalent transformations. The quotient records only that they belong to the same class. It forgets the multiplicity and topology of the connecting maps.

Suppose a family of presentations develops enhanced symmetry at a special parameter value. The quotient may represent this value as an ordinary point even though the local structure is singular.

Suppose equivalences around a loop compose to a nontrivial automorphism. The quotient collapses the loop and loses the holonomy.

These losses matter because symmetry, holonomy, phase, degeneracy, and topology are not merely decorative details. They can affect observable physics.

The insufficiency of an ordinary quotient can be expressed schematically.

A quotient set remembers:

which presentations represent the same meaning.

It forgets:

how the presentations are identified;

how many identifications exist;

whether identifications can be deformed into one another;

which self-equivalences preserve a presentation;

how transformations compose;

and whether higher obstructions occur.

Two systems can therefore have isomorphic quotient sets while possessing very different physical structures.

For example, imagine two groupoids with the same collection of equivalence classes. In the first, every object has only the identity automorphism. In the second, every object has a nontrivial symmetry group.

Their quotient sets may be identical, but their statistical weights, quantum phases, conserved structures, and response to transport may differ.

The ordinary quotient treats equivalence as erasure. The equivalence-first framework treats equivalence as structure.

This distinction is fundamental.

If physical identity is constituted by equivalence, then removing the equivalences after identifying the classes discards part of the identity itself.

The appropriate replacement must preserve not only the classes but also the transformations internal to those classes.

4.5 The Homotopy Quotient

A homotopy quotient retains the action and transformation structure that an ordinary quotient forgets.

Let a group or groupoid E act on the presentation space P. The ordinary quotient identifies points lying on the same orbit:

P / E.

The homotopy quotient, written schematically as

P // E,

retains the equivalence data associated with the action.

The notation

//

signals that the result is not merely a set of orbits. It is a groupoid, stack, or higher object containing the orbits together with stabilizers, loops, and coherent transformation data.

In Univalent Relational Dynamics, the physical moduli object is therefore proposed to have the form

M_phys = P // E_phys,

where E_phys is the admissible physical equivalence structure.

The ordinary quotient asks:

Which presentations are equivalent?

The homotopy quotient also asks:

Which equivalences connect them?

What symmetries remain at each presentation?

How do equivalences compose?

What loops and higher paths exist?

This richer construction is particularly important when the action is not free.

An action is free when no nontrivial transformation leaves a presentation fixed. If the action is free and regular, the ordinary quotient may behave smoothly.

If some p has a stabilizer

Stab(p) = {e in E_phys : e(p) = p},

then the quotient may possess singular or orbifold-like structure. The homotopy quotient retains Stab(p) as part of the point [p].

Physically, enhanced stabilizer structure can signal additional symmetry. Such points may correspond to special phases, degenerate configurations, vacuum sectors, fixed points, or critical configurations.

A homotopy quotient can also retain loop information. A closed sequence of equivalences may begin and end at the same presentation while composing to a nontrivial automorphism.

The resulting loop is invisible in the quotient set but visible in the homotopy quotient.

This makes the homotopy quotient a natural setting for holonomy and topological phase.

At higher levels, paths between equivalences and coherence among those paths are retained as well. The moduli object becomes an infinity-groupoid or higher stack rather than a conventional space.

This leads to a central proposal:

The physical world is not a quotient of presentations after equivalence is forgotten. It is a homotopy quotient in which equivalence remains part of the object.

Reality is not P.

Reality is not merely P/~.

Reality is the structured quotient

P // E_phys.

4.6 Fibers of Representation

Consider again the projection

q: P -> M.

For a physical meaning m in M, the fiber

F_m = q^-1(m)

contains all presentations realizing m.

The fiber is not simply a collection of redundant copies. Its structure measures how one physical identity can appear.

A large fiber means that many presentations correspond to the same meaning.

A small fiber means that the meaning is represented in relatively few ways.

A disconnected fiber may contain distinct families of presentations that are physically identified only through transformations not visible within the local topology of P.

A fiber with nontrivial loops contains presentation changes that return to the starting description while carrying nontrivial transformation history.

A fiber with a large automorphism group corresponds to a highly symmetric physical meaning.

The geometry of F_m may therefore influence physics.

One can imagine assigning a presentation multiplicity to m:

N(m) = number of presentations in F_m

for a finite system, or

Vol(F_m)

for a continuous system.

This quantity resembles the multiplicity underlying statistical entropy:

S(m) = k_B log Vol(F_m).

However, simple counting may overcount highly symmetric presentations. If a presentation has many automorphisms, several formal descriptions may correspond to the same internal symmetry orbit.

A refined measure may require division by automorphism volume or the use of homotopy cardinality.

For a finite groupoid, a schematic homotopy cardinality is

|G|_h = sum over equivalence classes [x] of 1 / |Aut(x)|.

This weighting recognizes that a highly symmetric object contributes differently from an object with trivial automorphism structure.

Fibers also matter dynamically.

A process may move within F_m, altering presentation while preserving physical identity.

A process may move from F_m to F_n, producing genuine physical change.

A bifurcation may split one fiber into several sectors.

A symmetry-breaking transition may reduce the automorphism structure of a fiber.

A measurement may map a rich fiber to a coarser set of classical outcomes.

A gauge fixing may select one representative from each fiber, when such a selection is possible.

The inability to choose representatives globally may itself indicate nontrivial topology.

Thus the fiber is not disposable redundancy. It contains the internal geometry of representation.

In an equivalence-first theory, one physical meaning is not a featureless point. It is a structured family of presentations connected by identity-preserving transformations.

4.7 Automorphism Groups

For a presentation x, the automorphism group is

Aut(x) = Equiv_phys(x,x).

Its elements are admissible self-equivalences of x.

The identity element

id_x

performs no change.

Other automorphisms transform the presentation while preserving its physical identity.

Aut(x) is important because self-equivalence is the mathematical form of symmetry.

If x represents a geometric object, Aut(x) may contain rotations, reflections, translations, or more general diffeomorphisms preserving its structure.

If x represents a field configuration, Aut(x) may contain gauge transformations leaving observables unchanged.

If x represents a quantum system, Aut(x) may contain unitary or projective symmetries.

If x represents a categorical object, Aut(x) may include natural or higher equivalences.

The automorphism structure can classify physical identity more precisely than a list of intrinsic properties.

Two presentations may share the same scalar observables while differing in their symmetry groups. Their physical behavior under perturbation, coupling, or transport may therefore differ.

The automorphism group also provides a possible foundation for charge.

Suppose a field or excitation transforms under a representation

rho: Aut(x) -> GL(V).

The transformation law of the excitation under self-equivalences defines a charge-like label.

From this viewpoint, charge is not an additional substance attached to an object. It describes how the object responds to the automorphism structure of its physical identity.

Different representations correspond to different transformation behaviors.

The trivial representation corresponds to invariance.

Nontrivial representations correspond to phase changes, rotations in an internal space, or more complicated responses.

Automorphism groups may vary across the moduli structure. At generic points, Aut(x) may be small. At special points, it may enlarge.

Such enhancement can signal symmetry restoration, phase transition, degeneracy, or critical behavior.

The stabilizer of a presentation is therefore not merely mathematical redundancy. It can determine local structure in the moduli space and influence physical weighting.

In path integrals and statistical sums, configurations with nontrivial automorphisms require special treatment because naive counting counts equivalent configurations repeatedly.

A schematic partition function over groupoid sectors may take the form

Z = sum over [x] of exp(-S[x]) / |Aut(x)|

for finite automorphism groups.

In continuous cases, the group size is replaced by a suitable group volume or gauge-fixed measure.

The presence of Aut(x) therefore affects symmetry, classification, probability, and dynamics.

An equivalence-first ontology naturally treats automorphism groups as part of what an object is.

4.8 Homotopy Groups as Physical Invariants

The homotopy structure of the physical moduli object records increasingly high levels of relational organization.

Let M_phys be the relevant higher moduli object, and let x be a chosen presentation or base point.

The zeroth homotopy set

pi_0(M_phys)

records connected components.

Two presentations lie in the same component if a path of admissible transformations connects them. Distinct components represent disconnected physical sectors.

These sectors may correspond to different phases, topological classes, conserved charges, vacuum sectors, or superselection sectors.

The first homotopy group

pi_1(M_phys,x)

records loops based at x, modulo continuous deformation.

A loop is a sequence of transformations that begins and ends at x.

A nontrivial element of pi_1 means that the loop cannot be continuously contracted to the identity.

Physically, such loops may represent holonomy, winding, phase accumulation, flux, or stable cyclic transformation.

The second homotopy group

pi_2(M_phys,x)

records closed surfaces of transformations. These may classify higher fluxes, monopole-like structures, relations among loops, or extended defects.

Higher homotopy groups

pi_n(M_phys,x)

record higher-dimensional coherence and obstruction.

The interpretation can be summarized schematically:

pi_0: disconnected physical sectors;

pi_1: symmetry loops, winding, or holonomy;

pi_2: relations among loops or surface-like charges;

pi_3: higher coherence or volume-like topological structure;

pi_n: generalized defects and higher conserved structures.

These groups are invariant under continuous deformation. They therefore provide stable classifiers of relational structure.

A particle-like object may arise as a localized nontrivial homotopy class. It persists because no allowed local deformation can remove it without crossing a singular configuration or changing boundary conditions.

A defect is therefore not necessarily a material object inserted into a medium. It may be a failure of the equivalence structure to become globally trivial.

This gives a precise form to the claim that matter may emerge from obstruction.

Suppose a local equivalence field can be trivialized in small regions but not globally. The obstruction is detected by a homotopy or cohomology class. That class can remain invariant under local perturbations.

The resulting object behaves as a persistent physical entity.

Homotopy invariants are particularly compatible with Univalent Relational Dynamics because they classify not isolated presentations but patterns of equivalence among presentations.

They measure what remains unchanged under continuous transformation.

This leads to a hierarchy of physical ontology.

A presentation is a local description.

An equivalence class is a physical meaning at set-level resolution.

A groupoid records the transformations constituting that meaning.

A homotopy quotient records stabilizers, loops, and higher relations.

Homotopy groups classify persistent obstructions and sectors within that relational structure.

The progression may be written as

P -> P/~ -> P//E -> higher invariants.

Each step retains more of the structure by which identity is formed.

The ordinary state space is therefore only one truncation of a richer object.

The central conclusion of this section is that physical meaning cannot, in general, be represented adequately by quotienting away descriptive redundancy and forgetting the transformations that produced the quotient.

A complete physical identity includes:

its presentations;

the equivalences among them;

the automorphisms preserving them;

the paths connecting alternative equivalences;

the loops formed by composition;

and the higher obstructions preventing global trivialization.

The proper arena for equivalence-first physics is therefore not a set of states but a moduli structure enriched by groupoid, homotopy, and higher-categorical data.

Reality is not merely the collection of equivalence classes.

Reality includes the structure of equivalence itself.

5. Dynamics as Transport and Extremization of Coherence

5.1 From Trajectories to Transformations

Conventional dynamics describes a physical system by a trajectory through a state space. If X is the state space, the system is represented by a map

x: T -> X,

where T is usually time and x(t) is the state at time t.

This formulation presumes that the identity of the system is already known. The trajectory merely records where that system is located in state space at each moment. The phrase “the same system at a later time” is treated as conceptually prior to the mathematical evolution.

Univalent Relational Dynamics reverses this order. It does not assume that the states x(t1) and x(t2) belong to the same persistent entity. Their common identity must be established by a transformation connecting them.

Instead of beginning with a path of states, the theory begins with a family of processes

T(t1,t2): x(t1) -> x(t2).

These maps express how a presentation at one stage is carried into a presentation at another. They must satisfy a compositional condition:

T(t2,t3) o T(t1,t2) ~= T(t1,t3).

The symbol ~= indicates that the two composite transformations need not be literally identical as presentations. They must be coherently equivalent.

The primitive dynamical object is therefore not the ordered list

x(t1), x(t2), x(t3), ...

but the network

x(t1) --T12--> x(t2) --T23--> x(t3),

together with all higher identifications that make the transports consistent.

A trajectory is recovered only after the process network has been projected onto a space of state presentations.

This distinction matters because two sequences may pass through the same collection of states while differing in their transformation histories. In the presence of geometric phase, gauge holonomy, topology, memory, or path dependence, the route cannot be discarded without losing physical information.

State-based dynamics asks:

Where is the system?

Equivalence-first dynamics asks:

How is the system’s identity being carried, and what structure is preserved during that transport?

This permits a broader account of change. Some transformations preserve identity. Others alter it. Some are reversible equivalences. Others are irreversible processes. Some compose without obstruction. Others accumulate curvature, phase, or inconsistency.

Dynamics becomes the study of structured transformation rather than motion among already individuated states.

5.2 Identity Transport

Let x and y be local presentations associated with two contexts, locations, times, scales, or observational frames. Identity transport is a map

T_gamma: F_x -> F_y,

where gamma is a path or process connecting the contexts and F_x and F_y are the corresponding local structures.

The role of T_gamma is to determine what it means for a physical entity described at x to be recognized at y.

In differential geometry, parallel transport carries vectors between tangent spaces. In gauge theory, a connection transports internal degrees of freedom. In quantum mechanics, unitary evolution transports amplitudes. In general relativity, transport along spacetime curves compares local frames.

Univalent Relational Dynamics interprets these as special cases of a common principle:

Physical persistence requires a rule for transporting identity between contexts.

The transport map need not preserve every descriptive feature. Coordinates, phases, bases, and local decompositions may change. What must be preserved is the equivalence structure defining the entity.

Suppose a local presentation contains an object E_x. Transport produces

T_gamma(E_x) = E_y.

The result E_y need not be numerically identical to E_x. It must instead be connected to it by the admissible equivalence determined by gamma.

Transport therefore has two aspects:

it carries structure,

and it certifies continuity of identity.

If no admissible transport exists, the presentations may belong to disconnected sectors. If transport exists but depends on the chosen path, the system possesses nontrivial geometry or topology. If transport fails to preserve the relevant structure, identity has changed rather than merely moved.

A transport law must also respect composition. If gamma1 connects x to y and gamma2 connects y to z, then

T_(gamma2 o gamma1) ~= T_gamma2 o T_gamma1.

The equality is generally coherent rather than literal because different descriptions of the composite path may differ by higher equivalence.

This provides the first dynamical expression of univalent identity. A physical object is not one static presentation repeated over time. It is the coherent transport class connecting its successive presentations.

5.3 Coherence Around Loops

The simplest test of transport consistency is to move around a closed loop.

Let gamma be a loop based at x:

gamma: x -> x.

Transport along gamma produces

T_gamma: F_x -> F_x.

If the transport is completely trivial, then

T_gamma ~= id_x.

However, a loop can return to the same presentation while producing a nontrivial automorphism:

T_gamma != id_x.

This discrepancy is a form of holonomy. It records the accumulated effect of transport around a closed path.

In Euclidean space with a flat connection, transporting a vector around a sufficiently simple loop returns it unchanged. On a curved surface, the vector may return rotated. In gauge theory, an internal state may acquire a phase or group transformation. In quantum theory, a cyclic evolution may produce a geometric phase.

These examples share a common structure:

the endpoint presentation is unchanged,

but the path of identification leaves a measurable residue.

Univalent Relational Dynamics interprets such residue as a failure of identity transport to close trivially.

Define a loop-defect operator

H_gamma = T_gamma o id_x^-1.

Since id_x^-1 = id_x, this reduces formally to

H_gamma = T_gamma.

The physically relevant question is whether H_gamma belongs to the trivial homotopy class or represents a nontrivial automorphism.

For small loops, the departure from identity can be associated with curvature. Schematically,

T_gamma ~= id + F(area) + higher-order terms,

where F measures the local obstruction to flat transport.

Curvature is therefore not introduced primarily as bending of a preexisting space. It is defined relationally as the failure of identity transport to be path-independent.

This interpretation applies to internal and external geometry alike.

Gauge curvature measures failure of internal-state transport to close.

Spacetime curvature measures failure of frame transport to close.

Topological flux measures failure of global transport to trivialize.

A quantum phase measures failure of amplitude transport to return literally to its initial presentation.

The loop is therefore a fundamental diagnostic of physical structure. It distinguishes a locally coherent but globally nontrivial world from one in which all equivalences can be flattened simultaneously.

5.4 Coherence Cost

Not all equivalences are equally easy to realize. Some require little action, energy, information, or geometric distortion. Others require large changes or may be dynamically forbidden.

To express this, assign a coherence cost

C(e)

to an admissible transformation e.

The cost may depend on several features:

the magnitude of local change,

the complexity of the transformation,

the length of the transport path,

the degree of mismatch between neighboring presentations,

the energy required to preserve identity,

the curvature generated by composition,

or the information needed to specify the equivalence.

The function C need not be a conventional distance, but it should encode how difficult it is to maintain coherent identity across a transformation.

A minimal set of desirable properties is

C(e) >= 0,

C(id_x) = 0,

and, for composable transformations,

C(g o f) <= C(f) + C(g),

at least in an effective or coarse-grained regime.

The final inequality states that the optimal direct cost cannot exceed the cost of a chosen two-step route. It resembles a triangle inequality.

If inverse transformations have equal cost, then

C(e^-1) = C(e).

This symmetry may hold for reversible transport but fail for irreversible processes.

The coherence cost allows relational proximity to be defined without assuming a background metric. Two presentations are close when a low-cost equivalence connects them. They are distant when every admissible identification is expensive.

A geometry may therefore emerge from cost:

d(x,y) = inf over e:x->y of C(e).

This quantity is the least cost of coherently transporting identity from x to y.

The cost function can also distinguish descriptive equivalence from physical dynamics. A pure gauge transformation may have zero physical cost even if its coordinate expression is complicated. A genuine excitation may carry positive cost because it changes invariant structure.

Coherence cost is therefore not merely computational complexity. It is a candidate physical primitive from which action, distance, mass, and interaction strength may eventually be derived.

5.5 A Relational Action Principle

A physical theory requires a rule selecting which transformation structures are realized. In conventional mechanics and field theory, this role is played by the action.

Univalent Relational Dynamics introduces an action over equivalence and process data rather than over isolated state trajectories.

Consider a discrete network with presentations x_i at vertices and transport maps

T_ij: x_i -> x_j

on edges.

A schematic relational action is

S[T,x] =
sum over edges (i,j) of C(T_ij)

- lambda sum over loops l of K(T_l,id)
- mu sum over vertices i of B(x_i).

The first term measures the cost of local transport.

The second term measures the failure of composed transport around loops to agree with the identity.

The third term measures local maintenance, boundary, compatibility, or excitation cost.

The quantity

T_l = T_in,i1 o ... o T_i2,i1 o T_i1,in

is the total transport around loop l.

The function K compares T_l with the identity. A simple choice might be

K(T_l,id) = ||T_l - id||^2,

when the transformation space supports a norm. More generally, K may be defined through group distance, categorical divergence, homotopy class, trace, phase, or representation-theoretic invariant.

The action can include higher-coherence terms. For example, if alternative compositions of three or more transformations differ, an associativity-defect penalty may be introduced:

S_higher = eta sum over coherence cells c of A(c).

The complete action becomes

S_total = S_local + S_loop + S_boundary + S_higher.

Classical dynamics may arise by extremizing the action:

delta S_total = 0.

Quantum dynamics may arise by summing over relational structures:

Z = integral exp(iS_total/hbar) D[T] D[x].

The objects being varied are no longer merely paths of states. They include transport maps, equivalence witnesses, local presentations, higher coherence data, and possibly the relational complex itself.

This allows geometry and matter to become dynamical together. Changing the transport maps alters the effective geometry. Changing the local equivalence types alters the matter content. Changing the network alters adjacency and causal structure.

The action is therefore not defined on a fixed arena containing objects. It is defined on the relational architecture from which arena and object may jointly emerge.

5.6 The Principle of Maximal Coherent Equivalence

A candidate foundational principle of the theory is:

Realized structures maximize coherent equivalence while minimizing the cost required to maintain it.

This statement is not equivalent to saying that everything should become identical. Complete equivalence would erase all distinctions and produce no structure. Complete inequivalence would produce isolated fragments with no persistent objects, geometry, or law.

Physical organization may require an intermediate regime.

Local presentations must be equivalent enough to form persistent objects and coherent geometry.

They must remain distinct enough to support gradients, boundaries, defects, charges, and dynamics.

The variational balance can be represented schematically as

S_eff = coherence cost + obstruction cost - equivalence benefit.

One possible finite form is

S_eff =
alpha sum_edges C(T_ij)

- beta sum_loops K(T_l,id)
- gamma N_eq,

where N_eq measures the number or weight of identity-preserving connections.

A system lowers S_eff by increasing coherent equivalence, but only when the required transport and compatibility costs do not become too large.

This principle gives a new interpretation to stable structure.

A stable object is a region in which many transformations preserve a common identity at relatively low cost.

A boundary is a region where the cost of extending that identity increases.

A defect is a region where coherence cannot be completed globally.

An interaction is a redistribution of equivalence and obstruction.

A phase transition is a reorganization of the network of low-cost equivalences.

The principle may also explain why symmetry and symmetry breaking coexist. Symmetry increases equivalence among presentations, but a perfectly symmetric configuration may be unstable or unable to support differentiated structure. Broken symmetry can reduce local equivalence while creating stable sectors, defects, and observables.

The physically realized world may therefore extremize not equivalence alone but a tradeoff among equivalence, complexity, stability, and obstruction.

5.7 Equivalence Geodesics

Given two presentations x and y, consider all admissible transformation paths connecting them:

e: x -> y.

The equivalence geodesic is the path minimizing coherence cost:

e* = arg min over e:x->y of C(e).

The corresponding relational distance is

d(x,y) = C(e*).

This construction generalizes the geometric notion of a shortest path. The path is not necessarily embedded in a preexisting space. Instead, the space and its metric are reconstructed from the cost of transformation.

An equivalence geodesic may represent:

the least-action evolution between two physical configurations,

the most efficient transport of identity,

the minimum-energy interpolation between phases,

the shortest proof of equivalence between structures,

the optimal gauge transformation,

or the minimal deformation connecting two presentations.

The geodesic equation should arise from variation of the cost functional.

If the path is parameterized by s and described locally by variables q(s), one may define

C[e] = integral L(q,dq/ds) ds.

The extremal path satisfies an Euler-Lagrange-type equation:

d/ds (partial L / partial q_dot)

- partial L / partial q
  = 0.

The novelty lies in the interpretation. The variables q need not represent the position of a material object. They may parameterize the changing equivalence data required to carry identity.

Geodesic deviation would then describe how nearby identity transports diverge. Curvature would measure the sensitivity of equivalence paths to initial conditions and loop structure.

The equivalence-geodesic concept also permits multiple geodesics between the same endpoints. Distinct minimal paths may interfere quantum mechanically or define different topological sectors.

A unique geodesic indicates essentially unambiguous identification.

Several disconnected geodesic families indicate multiple inequivalent ways of establishing sameness.

No geodesic indicates that the two presentations do not belong to the same admissible physical component.

5.8 Continuum Limits

A discrete relational network is useful for definition and computation, but established physics is usually expressed through continuum fields. A central challenge is therefore to derive continuum dynamics from increasingly dense equivalence structures.

Suppose presentations occupy vertices of a graph or higher complex. Neighboring vertices i and j carry transport maps T_ij.

For sufficiently small separations, write

T_ij ~= exp(A_mu dx^mu),

where A_mu is a local connection-like object.

Transport around an infinitesimal loop yields

T_loop ~= exp(F_mu_nu dx^mu dx^nu),

where

F_mu_nu =
partial_mu A_nu

- partial_nu A_mu
- [A_mu,A_nu].

The quantity F_mu_nu measures the lowest-order failure of local transports to commute and close trivially.

A discrete loop-coherence term can then approach a continuum curvature action:

sum_loops K(T_loop,id)
-> integral Tr(F_mu_nu F^mu_nu) dV.

Similarly, a local mismatch between neighboring presentations can approach a covariant-gradient term:

sum_edges C(T_ij)
-> integral ||D_mu phi||^2 dV,

where phi represents local presentation data and

D_mu phi = partial_mu phi + A_mu phi

is the covariant derivative.

A local maintenance term becomes a potential:

sum_vertices B(x_i)
-> integral V(phi) dV.

The continuum relational action may therefore take the schematic form

S =
integral [
a ||D phi||^2

- b ||F||^2
- V(phi)
- higher-coherence terms
  ] dV.

These expressions resemble familiar field theory, but their interpretation is different.

phi specifies local modes of presentation.

A specifies rules of identity transport.

D phi measures the failure of neighboring presentations to remain coherently related.

F measures obstruction to path-independent transport.

V measures the local cost of sustaining a chosen equivalence type.

The continuum limit must not simply assume a manifold. Ideally, the manifold, measure dV, dimension, and signature should emerge from the large-scale organization of the underlying relational network.

A successful derivation would show how smooth geometry appears when equivalence transport varies slowly and the network satisfies appropriate locality, regularity, and scaling conditions.

5.9 Conservation from Coherent Symmetry

Conservation laws arise when the action remains invariant under continuous transformations. In conventional physics, this relationship is formalized by symmetry principles.

Univalent Relational Dynamics interprets symmetry as self-equivalence of the relational structure.

Let a family of transformations g_epsilon act on the local presentations and transports. If

S[g_epsilon(T,x)] = S[T,x]

for all sufficiently small epsilon, then the action possesses a continuous equivalence symmetry.

The corresponding conserved quantity measures what remains unchanged as the system moves along that family of equivalent descriptions.

In a continuum model, if the Lagrangian L is invariant under

q -> q + epsilon Delta q,

then a conserved current J^mu satisfies

partial_mu J^mu = 0.

The equivalence-first interpretation is:

a conserved current records the transport of a quantity protected by coherent self-equivalence.

Energy conservation may correspond to invariance under temporal translation in an emergent time structure.

Momentum conservation may correspond to invariance under emergent spatial translation.

Angular momentum may correspond to rotational self-equivalence.

Charge conservation may correspond to invariance under an internal automorphism group.

Topological charges may be conserved because no continuous local deformation can change the relevant homotopy class.

Conservation is therefore not the persistence of a substance. It is the persistence of an invariant under an admissible family of transformations.

This interpretation is especially natural when the conserved quantity is not localized in an ordinary material object. Flux, winding, phase, and topological number are properties of relational structure rather than contents stored inside particles.

The dynamical picture developed in this section can now be summarized.

States become local presentations.

Trajectories become composable transport maps.

Persistence becomes coherent identity transport.

Curvature becomes failure of loop closure.

Distance becomes minimum coherence cost.

Action becomes a functional of transformations and obstructions.

Stable structure arises from an extremum between equivalence and the cost of maintaining it.

Conservation arises from coherent self-equivalence.

The foundational dynamical question is therefore no longer simply

How does a state move?

It becomes

How is identity transported, what does that transport cost, where does it fail to close, and which relational structures extremize the resulting coherence action?

6. Emergent Geometry, Spacetime, and Gravitation

6.1 Relational Proximity

Conventional geometry begins with a space whose points already possess relations of nearness, distance, angle, and dimension. Physical objects are then placed within that space. Univalent Relational Dynamics reverses this order. It does not assume that two physical contexts are related because they occupy nearby points. It proposes that they count as nearby because their structures can be coherently identified at low cost.

Let x and y denote local physical contexts. These contexts may represent measurement regions, local algebras, informational neighborhoods, field patches, or nodes in an underlying relational complex. Suppose there exists an admissible transport

e: x -> y.

The existence of e means that at least some physical identity can be carried from x to y. Its coherence cost C(e) measures the difficulty of performing that identification.

The basic relational principle is:

x and y are physically near when a low-cost coherent transport connects them.

This definition does not require x and y to be embedded in a preexisting manifold. Their proximity is inferred from the structure and cost of the morphisms between them.

If many low-cost transports connect x and y, they are strongly adjacent.

If only expensive transports connect them, they are relatively distant.

If no admissible transport exists, they may belong to disconnected physical sectors.

Relational proximity is therefore richer than a binary adjacency relation. It may depend on:

the number of available transports;

their minimum cost;

their stability under perturbation;

their compositional compatibility;

their causal direction;

and their higher coherence.

Two contexts may be close for one type of structure and distant for another. Internal phase information may be transported easily while massive matter is transported only at high cost. Classical observables may be strongly connected while quantum coherence is weakly connected. Geometry may therefore initially be typed or multidimensional, with different families of equivalence generating different effective notions of distance.

A single macroscopic geometry would emerge only when these transport costs become sufficiently regular and mutually compatible.

The principle can be summarized by reversing the usual logical order:

Points are not connected because they are near.

They are near because coherent low-cost transformations connect them.

6.2 Distance from Equivalence Cost

Given two contexts x and y, define the candidate distance

d(x,y) = inf over e:x->y of C(e).

The infimum is taken over all admissible transports connecting x to y.

This construction interprets distance as the least coherence cost required to carry physical identity between two contexts.

For d to behave as an ordinary metric, several conditions are desirable:

d(x,y) >= 0,

d(x,x) = 0,

d(x,y) = d(y,x),

and

d(x,z) <= d(x,y) + d(y,z).

The first condition follows if coherence cost is nonnegative.

The second follows if the identity transport has zero cost:

C(id_x) = 0.

The triangle inequality follows if transport costs are subadditive:

C(g o f) <= C(f) + C(g).

Symmetry is less automatic. If every admissible transport has an inverse of equal cost,

C(e^-1) = C(e),

then

d(x,y) = d(y,x).

However, fundamental processes may be directional. A decay, measurement, erasure, or causal signal may not possess an equally available inverse. The resulting relational structure may define an asymmetric distance, a directed metric, or separate forward and backward costs:

d_+(x,y) != d_+(y,x).

This asymmetry may become relevant to causal order and the arrow of time.

The separation condition

d(x,y) = 0 implies x ~= y

also requires care. Distinct presentations may have zero distance because they are physically equivalent. Thus d is naturally a pseudometric on presentations. It becomes a genuine metric only after passing to physical equivalence classes.

If [x] and [y] denote equivalence classes, one may define

d_phys([x],[y]) = inf over x' in [x], y' in [y] of d(x',y').

A successful theory must ensure that this result is independent of arbitrary presentation choices.

The geometry of physical space would then arise from the metric structure of the quotient or homotopy quotient, rather than from the raw presentation space.

The cost function may depend on scale. At microscopic resolution, the geometry may be irregular, discrete, or nonlocal. At larger scales, repeated composition of local transports may produce a smooth effective metric.

The usual line element

ds^2 = g_mu_nu dx^mu dx^nu

would then be interpreted as the continuum approximation to accumulated infinitesimal coherence cost.

The metric tensor g_mu_nu would not be primitive. It would summarize how difficult it is to transport identity in different local directions.

6.3 Topology Before Metric

A numerical metric is not the only way to define geometry. A weaker structure can arise from the pattern of possible equivalences before any exact cost is assigned.

For each context x and threshold epsilon, define a relational neighborhood

N_epsilon(x) = {y : there exists e:x->y with C(e) < epsilon}.

These neighborhoods specify which contexts can be reached from x by sufficiently low-cost transformations.

If the neighborhoods satisfy appropriate consistency properties, they generate a topology. Open regions are collections of contexts stable under sufficiently small coherent transport.

This provides a natural order of construction:

first, admissible connection;

second, neighborhood;

third, topology;

fourth, metric;

fifth, smooth geometry.

The theory therefore does not need to assume a differentiable manifold at its foundation.

A primitive relational network may determine which nodes belong to common neighborhoods. As the network becomes dense, these neighborhoods may approximate open sets in a continuum.

Topological properties can emerge before numerical distance. One can ask whether the relational structure is connected, whether it contains holes, whether loops are contractible, and whether local patches can be glued globally.

These properties may remain stable even when precise costs fluctuate.

This suggests that topology may be more fundamental than metric geometry. Metric values can change continuously while connectivity and homotopy class remain unchanged.

A physical phase could therefore be characterized first by the topology of its equivalence network. Metric properties would describe finer energetic or dynamical details within that phase.

The local-to-global axiom becomes crucial here. A collection of local neighborhoods does not automatically define a coherent global space. Transition maps must satisfy overlap conditions. Nontrivial gluing may generate global topology even when every local patch is simple.

Spacetime may therefore arise not as a set of primitive points but as the moduli object obtained by gluing local contexts through admissible equivalences.

The points of the emergent space are then secondary. They represent equivalence classes of local relational data.

6.4 Metric Structure from Transport

To obtain an effective physical metric, the relational cost must possess sufficient regularity.

Suppose the local contexts can be parameterized by coordinates x^mu in a continuum limit. Consider an infinitesimal displacement

dx^mu.

The minimum coherence cost between neighboring contexts may be expanded as

C(x,x+dx)^2 ~= g_mu_nu(x) dx^mu dx^nu.

The coefficients g_mu_nu define an effective metric tensor.

This relation should not be interpreted as inserting a metric by hand. It defines g_mu_nu operationally from the local second-order behavior of transport cost.

If transport in one direction is expensive, the corresponding metric coefficient is large.

If transport is inexpensive, the coefficient is small.

If different directions interact, off-diagonal terms appear.

The effective dimension is related to the rank of the local quadratic form. If only d independent directions carry nonzero transport cost, then the emergent geometry has local dimension d, even if the underlying presentation space uses more coordinates.

This distinction is important. The number of descriptive variables need not equal the number of physical dimensions. Rank deficiency can indicate gauge redundancy, constrained motion, or emergent lower-dimensional structure.

A positive-definite metric would describe a purely spatial or Euclidean geometry. A spacetime metric requires indefinite signature. That signature cannot be obtained from an ordinary nonnegative cost without additional structure.

One possibility is that the primitive theory distinguishes reversible equivalence directions from directed process directions. Reversible transport contributes spatial-like positive terms, while irreversible causal composition contributes a distinguished time-like direction.

The interval could then have the form

ds^2 = -c^2 dt^2 + h_ij dx^i dx^j.

The negative sign would encode a categorical distinction rather than an arbitrary convention: temporal separation would arise from directed process order, whereas spatial separation would arise from reversible equivalence cost.

Another possibility is that the metric emerges from a phase-valued or action-valued functional rather than a simple positive cost. Stationary phase could distinguish time-like, null, and space-like relations.

The derivation of Lorentzian signature remains a major open problem. An equivalence-first framework must do more than reproduce a generic distance. It must explain why large-scale physics exhibits one effective temporal dimension and several spatial dimensions.

6.5 Curvature as Failed Identity Closure

Curvature can be defined without beginning with a curved manifold. It can be recognized through the failure of local transport to close consistently.

Consider an infinitesimal loop based at x. Let its directed edges carry transports

T_12, T_23, T_34, T_41.

The total loop transport is

T_loop = T_41 o T_34 o T_23 o T_12.

If the relational structure is locally flat, then

T_loop ~= id_x

for sufficiently small contractible loops.

If

T_loop != id_x,

the loop retains a nontrivial residue. This residue measures curvature.

For a connection A_mu, one writes locally

T_loop ~= exp(F_mu_nu Sigma^mu_nu),

where Sigma^mu_nu is the oriented loop area and

F_mu_nu =
partial_mu A_nu

- partial_nu A_mu
- [A_mu,A_nu].

The curvature F_mu_nu measures the leading failure of transport in different directions to commute.

In an equivalence-first interpretation, curvature is not initially a property of a background surface. It is a property of the composition of identity transports.

The question is not first whether space bends. The question is whether one can transport a physical standard around a loop and return it unchanged.

If not, the relational field possesses curvature.

This definition unifies several structures.

In gauge theory, the transported object is an internal state, and curvature is gauge-field strength.

In differential geometry, the transported object is a tangent vector or frame, and curvature measures geometric holonomy.

In quantum theory, the transported object may be a state ray, and curvature may produce geometric phase.

In higher gauge theory, surfaces and volumes carry higher transports, and curvature includes higher-form components.

The same categorical pattern appears at every level:

local equivalences compose;

alternative compositions are compared;

failure of trivial closure defines curvature.

Higher curvature arises when not only edge transports but also surface-level and volume-level coherence fail to close.

A 2-connection may assign transport to surfaces. Its curvature measures the failure of surface transport to be independent of higher deformation.

Thus the geometry of reality may consist of a hierarchy of coherence failures, not only the ordinary curvature of lines transported around loops.

6.6 Gravitation as Coherence Geometry

General relativity describes gravitation through spacetime geometry. Matter and energy influence the metric, and the metric determines free motion.

Univalent Relational Dynamics proposes a deeper interpretation:

Gravitational geometry is the macroscopic organization of identity-transport cost.

A freely moving body follows a worldline along which its physical identity is transported with extremal coherence cost. In the continuum limit, this becomes a geodesic.

The conventional geodesic equation is

d^2 x^mu/dtau^2

- Gamma^mu_(alpha beta)
  (dx^alpha/dtau)(dx^beta/dtau)
  = 0.

In the equivalence-first framework, the connection coefficients Gamma^mu_(alpha beta) summarize how local standards of identity comparison vary from one context to another.

The object does not move along a curved path because an invisible gravitational force pulls it. It follows the path requiring the appropriate extremal transport through a nonuniform coherence structure.

Mass-energy would modify the local availability or cost of equivalences. Regions containing energy, momentum, pressure, or stress would reorganize how identity can be carried across neighboring contexts.

A schematic relational field equation might take the form

coherence curvature = kappa x source of coherence obstruction.

In a continuum approximation, this would need to reduce to an Einstein-like relation:

G_mu_nu + Lambda g_mu_nu = kappa T_mu_nu.

The left side describes curvature of emergent identity transport.

The right side describes matter, energy, momentum, and stress as sources altering that transport structure.

This interpretation suggests a mutual dependence:

matter is a stable pattern within the equivalence field;

geometry is the transport structure surrounding that pattern;

and gravitation is the self-consistent adjustment between the two.

Matter does not simply sit inside spacetime.

Spacetime does not merely act upon matter.

Both emerge from one relational coherence system.

A massive object could be a region in which equivalence transport has a persistent nontrivial cost. That cost alters neighboring transport paths. The resulting large-scale distortion appears as gravitational curvature.

This idea must eventually yield quantitative equations. It is not enough to rename curvature as coherence. The theory must derive an effective metric, specify its source terms, reproduce gravitational redshift, lensing, orbital motion, and wave propagation, and explain the equivalence of inertial and gravitational mass.

6.7 Equivalence and the Equivalence Principle

The equivalence principle of general relativity states, in one of its forms, that the local effects of a uniform gravitational field are indistinguishable from those of acceleration. Locally, freely falling frames remove gravitational force.

The univalent identity principle concerns a different notion of equivalence. It states that admissibly equivalent structures constitute the same physical identity.

The two principles should not be conflated, but they may be related.

A freely falling frame is a local presentation in which gravitational effects are minimized or transformed into inertial motion. Another frame may describe the same local physics using gravitational acceleration.

The equivalence between these descriptions suggests that inertial and gravitational presentations belong to one local physical identity structure.

In Univalent Relational Dynamics, the gravitational equivalence principle may emerge from the existence of local transformations that trivialize the connection along a chosen worldline.

At a point or along an infinitesimal region, one can choose a local frame in which

Gamma^mu_(alpha beta) ~= 0.

This does not eliminate curvature. Derivatives of the connection remain, and transport around finite loops may still be nontrivial.

The distinction mirrors the difference between local equivalence and global obstruction.

Gravity can be removed locally as a force-like presentation.

Gravity cannot generally be removed globally because the transport structure has curvature.

The equivalence principle therefore becomes an example of the broader rule:

Locally, one may choose presentations that simplify identity transport; globally, coherence obstruction remains invariant.

This interpretation may also clarify why inertial mass and gravitational mass are equal. If both measure the same underlying resistance or cost associated with transporting a coherence pattern, their equality is structural rather than accidental.

Inertial mass would quantify the cost of changing the transport of an identity pattern.

Gravitational mass would quantify how that pattern modifies and responds to the surrounding coherence geometry.

A successful theory should derive their equality from one common relational functional.

6.8 Spacetime Dimension

The dimensionality of spacetime is normally assumed at the outset. A fundamental theory should ideally explain why the large-scale world has its observed effective dimension.

In an equivalence network, dimension can be inferred from several relational properties.

One possibility is neighborhood growth. Let N(r) be the number of contexts reachable from x with cost less than r. If

N(r) ~ r^d

over some range, then d acts as an effective dimension.

A second possibility is spectral dimension. A diffusion process on the relational network may return to its origin with probability

P_return(t) ~ t^(-d_s/2).

The exponent defines a spectral dimension d_s.

A third possibility is local rank. If infinitesimal transport cost is governed by a quadratic form g_mu_nu, the number of nonzero independent directions defines the local metric dimension.

A fourth possibility is homological dimension, determined by the dimensions of nontrivial cycles and cells required to represent coherence structure.

These definitions need not agree at microscopic scales. The underlying network may be fractal, anisotropic, or scale-dependent. At large scales, they may converge toward a stable integer dimension.

The theory must also explain why one dimension is time-like.

One proposal is that spatial dimensions correspond to locally reversible families of equivalence, while time corresponds to directed composition of noninvertible processes.

Space-like variation permits reversible comparison:

x <-> y.

Time-like variation permits only an ordered process:

x -> y.

The number of spatial dimensions would then reflect the number of independent reversible transport directions. The single temporal dimension might arise from the global ordering required for consistent noninvertible composition.

Multiple time dimensions could generate causal ambiguity, unstable composition, or failure of a coherent partial order. A one-dimensional process order may be the minimal structure capable of supporting consistent causal sequencing.

These possibilities remain conjectural. The observed dimension must emerge from explicit models rather than verbal analogy.

A computational research program can test whether networks selected by coherence-extremization naturally produce stable dimensionality, and whether four-dimensional Lorentzian behavior is dynamically favored.

6.9 Lorentzian Structure and Causal Order

A relational theory must distinguish geometric adjacency from causal accessibility.

Two contexts may be structurally near but not causally connected. Conversely, a causal process may connect contexts through a directed chain whose reverse is forbidden.

Let

x <= y

mean that there exists an admissible future-directed process

f: x -> y.

This relation can define a causal order when it is transitive:

x <= y and y <= z imply x <= z,

and appropriately antisymmetric:

x <= y and y <= x imply x ~= y

for contexts that are causally identical.

The causal structure can therefore be constructed from the existence of directed morphisms rather than assumed from a time coordinate.

A light cone becomes the boundary between contexts accessible through causal transport and contexts connected only by space-like equivalence.

Null relations may represent transformations propagated at the maximal speed compatible with the local coherence structure.

The constant c would then characterize a universal conversion between temporal process cost and spatial equivalence cost.

In an emergent continuum, causal order and volume information may be sufficient to reconstruct much of the Lorentzian geometry. This suggests that the primary structure could be:

directed morphisms;

their compositional order;

and a measure of relational density.

The metric would then be derived rather than postulated.

Lorentz invariance would require that no preferred local presentation of causal structure be physically observable. Different inertial frames would be connected by admissible equivalences preserving the causal cone and interval structure.

Boost transformations would therefore belong to the local automorphism group of emergent spacetime.

At microscopic scales, exact Lorentz invariance might be fundamental, approximate, or emergent. A discrete relational network risks introducing a preferred frame. To avoid conflict with observation, the continuum limit must either erase this frame or construct the network in a way that preserves causal symmetry statistically or exactly.

This issue provides an important falsifiability test. Residual anisotropy, dispersion, or energy-dependent propagation could reveal failure of the emergent symmetry.

6.10 Background Independence

A background-dependent theory assumes a fixed arena in which physical fields evolve. The arena supplies points, neighborhoods, metric relations, and causal structure before matter is introduced.

Univalent Relational Dynamics aims for a stronger form of background independence. The network of contexts, the admissible transports, the cost structure, and the local presentation types are all dynamical.

There is no fixed space beneath the equivalences.

The pattern of equivalence is what produces space.

There is no external time parameter required at the deepest level.

The directed composition of processes is what produces temporal order.

There is no predetermined metric.

The cost of coherent transport produces effective distance.

There is no fixed curvature field imposed on matter.

Curvature is the failure of identity transport to close.

A fully background-independent model would sum or extremize not only over field values but also over relational architectures.

Schematically,

Z =
sum over complexes K
integral D[x] D[T] D[higher coherence]
exp(iS[K,x,T]/hbar).

Here K represents the underlying relational complex, x the local presentations, T the transport data, and the remaining variables the higher coherence structure.

Different choices of K may correspond to different candidate geometries, dimensions, topologies, and causal organizations.

The theory must prevent this freedom from becoming arbitrary. The action, consistency conditions, and observational constraints must strongly restrict which relational structures contribute.

Background independence also raises the problem of observables. Without a fixed spacetime point, a quantity such as “the value of a field at x” is not invariant. Observables must be relational.

One may ask for the value of one structure when another serves as a reference. One may define coincidences, correlations, scattering relations, boundary amplitudes, or invariant loop quantities.

The framework therefore predicts that fundamentally meaningful observables will be relational and descend through the full equivalence structure.

The emergent geometry developed in this section can be summarized in a chain of construction:

admissible transformations define relational adjacency;

transport cost defines proximity;

proximity defines neighborhoods;

neighborhoods define topology;

regular cost defines metric;

loop obstruction defines curvature;

directed composition defines causal order;

stable large-scale organization defines spacetime.

Gravitation then appears as the dynamics of this emergent transport geometry.

The central claim is not that conventional spacetime is unreal. It is that spacetime may be an effective macroscopic representation of a deeper structure whose primitive elements are equivalences, processes, coherence costs, and obstructions.

The conventional statement is:

Physical systems move through spacetime.

The equivalence-first statement is:

Spacetime is the large-scale geometry traced by the possible transport of physical identity.

The decisive test will be whether this conceptual reversal can be turned into a quantitative theory that recovers established relativistic physics while explaining why geometry, dimension, causal structure, and gravitation take the forms that they do.

7. Matter as Stable Coherence, Defect, and Representation

7.1 Objects as Persistent Relational Patterns

In conventional physics, matter is usually treated as something that occupies space, carries properties, and participates in interactions. A particle has mass, charge, spin, momentum, and location. A field has values at spacetime points. Composite objects are constructed from more elementary components.

Univalent Relational Dynamics proposes a different starting point. Matter is not first defined as substance. It is defined as a persistent pattern in the network of admissible transformations.

A physical object is a relational structure that can be transported, deformed, and observed while preserving a coherent identity type.

Let E denote such a pattern. Its persistence is expressed through a family of admissible transports

T_gamma: E_x -> E_y,

where x and y are different contexts and gamma is a process connecting them.

The object persists when the transported presentation remains equivalent to the original pattern:

T_gamma(E_x) ~= E_y.

The object therefore does not reside in any one presentation. It is distributed across the equivalence structure connecting its presentations.

This view explains why physical identity can survive substantial change. A body can move, rotate, accelerate, exchange energy, or be described in another frame while remaining identifiable. What persists is not every microscopic coordinate. What persists is a structured network of invariants, transformation laws, and compositional relations.

Matter becomes a stable solution to the problem of relational continuity.

A pattern qualifies as object-like when it satisfies several conditions:

it can be distinguished from its surroundings;

it supports coherent transport;

it possesses a stable automorphism structure;

it resists arbitrary deformation;

it participates consistently in composition and interaction;

and it remains identifiable across a suitable range of scales.

An object is therefore neither a featureless point nor a mere equivalence class. It is a region of relational structure with enough internal coherence to support persistence.

The boundary of an object marks a change in the cost or admissibility of extending that coherence. Inside the object, transformations preserve a common identity pattern. Across the boundary, additional structure, energy, or reconfiguration is required.

This suggests a relational definition of material localization:

Matter is localized where coherence is concentrated and where extension of identity across neighboring contexts encounters a nontrivial cost gradient.

The apparent solidity of matter may then arise from the stability and resistance of its equivalence structure rather than from an underlying indivisible substance.

7.2 Particles as Topological Defects

A particularly powerful form of persistence is topological persistence.

Suppose a local equivalence field can be trivialized in small regions. Within each region, the structure can be transformed into a standard reference form. However, when the local trivializations are combined around a closed loop, surface, or higher cycle, they may fail to agree globally.

This failure is a topological defect.

A particle-like excitation can therefore be modeled as a localized obstruction to globally trivializing the equivalence field.

Let F be an equivalence field defined on a region excluding a localized core. Around the core, transport defines a map into a symmetry or moduli space M. If the map belongs to a nontrivial homotopy class,

[gamma] in pi_n(M),

then the configuration cannot be continuously deformed into the trivial configuration without crossing a singular state or altering the boundary conditions.

The defect persists because local changes cannot remove the global obstruction.

This produces a precise form of matter without substance. The particle is not an additional object placed inside the field. It is a stable feature of the field's relational topology.

Examples of the general pattern include:

a loop surrounding a defect with nontrivial winding;

a sphere enclosing a monopole-like structure;

a vortex around which phase transport fails to close;

a domain wall separating disconnected equivalence sectors;

and a higher-dimensional defect classified by a higher homotopy group.

The relational interpretation is:

the defect exists because identity cannot be extended coherently through the enclosed region.

Its persistence follows from topology rather than from local rigidity alone.

This approach also clarifies particle creation and annihilation. A defect and an antidefect may appear together when the global topological charge remains conserved. They may annihilate when their combined obstruction becomes trivial.

A particle collision can then be described as a reorganization of defect classes within the equivalence field.

Stable particles correspond to protected topological or categorical structures.

Unstable particles correspond to coherence patterns that can decay into lower-cost configurations while preserving the relevant global invariants.

The particle spectrum of a theory would therefore be determined by the admissible defects of its underlying moduli structure.

A successful theory must identify a specific equivalence field whose stable defects reproduce the observed particle classes, quantum numbers, and interactions.

7.3 Mass as Equivalence Inertia

Mass measures resistance to acceleration and acts as a source of gravitational curvature. In an equivalence-first framework, both roles may arise from the cost of transporting a persistent coherence pattern.

Let E be an object-like equivalence structure and gamma a worldline or transport path. Assign a transport cost

C(E,gamma).

In a low-energy continuum regime, the cost may approach

C(E,gamma) ~= m integral ds,

where ds is the invariant path element and m is a coefficient characterizing the pattern.

The coefficient m is interpreted as equivalence inertia.

It measures how much action is required to preserve the identity of E while carrying it through changing contexts.

A more complex object generally contains a richer coherence structure. Transporting it requires maintaining more internal relations, phase conditions, symmetries, or topological constraints. Its transport cost may therefore be greater.

Mass is then not an amount of material substance. It is a measure of resistance to coherent re-embedding.

Acceleration changes the transport path of the object. To accelerate E, the surrounding relational structure must alter the way E is identified from one context to the next. The greater the coherence cost of this alteration, the greater the inertial mass.

This interpretation suggests

inertial mass = resistance to changing identity transport.

The same structure may also modify the surrounding geometry. A coherence pattern with nonzero maintenance cost changes the local network of admissible equivalences and therefore alters neighboring transport paths.

This suggests

gravitational mass = influence of a coherence pattern on the surrounding transport geometry.

If both arise from the same underlying functional C(E,gamma), the equality of inertial and gravitational mass becomes natural.

Mass could also depend on internal automorphism structure. A pattern with a large symmetry group may possess many equivalent internal presentations, reducing some transport costs while increasing others. Symmetry breaking may generate mass by restricting which local equivalences remain available.

A mass term in an effective action could therefore represent the energetic cost of preserving a pattern when perfect relational symmetry is absent.

The familiar expression

$E = mc^2$

would then relate rest energy to the minimum ongoing cost of sustaining a stable equivalence pattern against dissolution into the surrounding relational field.

Rest energy is the energetic signature of persistent coherence.

7.4 Rest Mass and Internal Coherence

Rest mass remains when translational motion is removed. It must therefore arise from structure internal to the object rather than from the cost of changing its external position alone.

In Univalent Relational Dynamics, rest mass is associated with the minimum cost of maintaining the object's internal equivalence organization.

Let E contain a network of internal relations indexed by a set of components or local contexts. The maintenance functional may be written schematically as

M(E) =
sum over internal links C(T_ij)

- lambda sum over internal loops K(T_loop,id)
- mu B(E).

The first term measures the cost of maintaining internal transports.

The second measures nontrivial internal curvature or coherence obstruction.

The third measures boundary and localization cost.

The rest mass is then related to the minimum of this functional over all presentations of the same physical identity:

m(E)c^2 = inf over E' ~= E of M(E').

This definition separates physical mass from descriptive complexity. A complicated presentation should not automatically have greater mass. The minimization is taken over equivalent presentations so that only invariant coherence cost remains.

A topological defect may retain nonzero rest mass because its obstruction cannot be removed by local transformation.

A composite object may have a rest mass different from the sum of the rest masses of its components because binding alters the total coherence cost.

If binding increases relational compatibility and lowers the total maintenance cost, energy is released and the composite mass decreases.

If internal excitation increases curvature or incompatibility, the rest mass increases.

This provides a relational interpretation of binding energy and internal energy:

energy changes when the cost of sustaining the equivalence structure changes.

The framework must eventually explain why specific particles possess specific masses. That requires a concrete model in which the minima of the coherence functional can be calculated and compared with observation.

Without such a derivation, mass as equivalence inertia remains an interpretation rather than a predictive result.

7.5 Massless Propagation

A massless excitation has no rest frame and propagates at the invariant speed of the emergent causal structure. In an equivalence-first framework, it can be interpreted as a coherence disturbance that requires no localized rest-maintenance cost.

A massless pattern does not exist as a stationary object whose internal identity must be sustained independently of propagation. Its identity is inseparable from its movement through the relational network.

Let W denote a propagating coherence mode. If

$M(W) = 0$

but the transport cost along a null path is nonzero, then W carries energy and momentum without possessing rest mass.

The excitation may be viewed as a self-propagating transformation of the equivalence field.

Its persistence lies not in a localized structure transported from point to point, but in a conserved relational waveform reproduced along successive contexts.

A massless excitation therefore satisfies a principle of propagation identity:

the pattern remains the same only through continued transport.

Stopping the pattern would destroy the form of identity that defines it.

This distinguishes massless and massive objects.

A massive object supports a persistent internal coherence structure even in a rest frame.

A massless excitation is a persistent transport structure with no rest-frame realization.

In an effective continuum theory, this distinction appears in the dispersion relations

$E^2 = p^2 c^2 + m^2 c^4$

for a massive mode, and

$E = pc$

for a massless mode.

The equivalence-first interpretation is that the mass term measures local maintenance cost, while the momentum term measures transport cost.

A symmetry-preserving relational mode may remain massless because no invariant local term can assign it a rest-maintenance cost.

Symmetry breaking, environmental coupling, or obstruction may convert such a mode into a massive excitation by creating a nonzero local coherence cost.

This offers a relational language for mass generation.

7.6 Charge as Automorphism Response

Charge is conventionally treated as a property carried by matter. In gauge theory, however, charge is fundamentally connected with how fields transform under a symmetry group.

Univalent Relational Dynamics makes this transformation behavior primary.

Let E be a physical identity pattern with local automorphism group

Aut(E).

A matter-like excitation may transform under a representation

rho: Aut(E) -> GL(V),

where V is a space of internal presentations.

The representation rho specifies how the excitation responds when the underlying physical identity is acted upon by self-equivalences.

Charge is therefore defined as automorphism response.

An uncharged excitation transforms trivially:

rho(g)v = v

for all g in Aut(E).

A charged excitation transforms nontrivially:

rho(g)v != v

for at least some g.

For a phase symmetry, one may have

rho(theta)v = exp(iq theta)v,

where q is the charge parameter.

The value q specifies how strongly the excitation responds to the local symmetry transformation.

This interpretation removes the need to regard charge as an invisible substance stored inside a particle. Charge is a relational law describing transformation behavior.

Electric charge, color charge, and other quantum numbers may correspond to representations of different automorphism groups.

The conservation of charge follows when the relational action remains invariant under the associated automorphism structure. Interactions must compose representations in a way that preserves the total transformation law.

If particles with charges q1 and q2 combine, the composite representation determines the total charge. In an Abelian case,

$q_{total} = q1 + q2$.

For non-Abelian symmetries, charges combine through tensor products and representation decomposition rather than simple addition.

Charge conjugation corresponds to passage to a dual or complex-conjugate representation.

Neutral composites can arise when nontrivial representations combine to contain the trivial representation.

The physical meaning of charge is therefore inseparable from composition.

It states not only how one object transforms, but how its transformation behavior combines with that of other objects.

7.7 Internal Symmetries

An internal symmetry changes a system's internal presentation without changing its spacetime location. In conventional gauge theory, these symmetries are represented by groups acting on fields.

In Univalent Relational Dynamics, internal symmetry groups arise as automorphism structures of local identity types.

Suppose each local context x carries a fiber F_x of internal presentations. The group

$G_x = Aut(F_x)$

contains transformations preserving the physical identity represented by that fiber.

A gauge choice selects one local presentation from F_x.

A gauge transformation selects another equivalent presentation.

A connection specifies how presentations in neighboring fibers are compared.

A gauge field therefore becomes a rule for transporting internal equivalence.

The gauge group is not added externally to a preexisting particle. It is part of the particle's local identity structure.

Different matter types correspond to different representations of G_x.

Interactions arise from the requirement that local transformations and transports remain coherent under composition.

The central research question is whether the observed internal symmetry groups can be derived rather than assumed.

A candidate theory must explain why the physically realized automorphism structure contains the particular groups and representations seen at accessible energies.

Possible sources include:

the automorphisms of a fundamental higher category;

the stabilizers of selected equivalence types;

the symmetries of a finite relational complex;

the structure of a higher topos;

or the topology of a moduli object.

Symmetry breaking occurs when the realized configuration is preserved by only a subgroup

H subset G.

The original automorphism structure remains in the laws, but the selected presentation or vacuum exhibits reduced symmetry.

The quotient

G/H

then describes directions of broken equivalence.

Excitations along these directions may generate massless modes, massive gauge fields, defects, or phase transitions depending on the dynamics.

The equivalence-first account therefore treats spontaneous symmetry breaking as a reorganization of which self-equivalences can be realized locally at low cost.

7.8 Antiparticles and Duality

An antiparticle has the same mass and spin as its associated particle but opposite additive charges and conjugate transformation behavior.

This structure suggests a natural categorical interpretation.

If a particle-like object is represented by E, its antiparticle may correspond to a dual object

E*.

The dual is characterized by evaluation and coevaluation processes:

ev: E* tensor E -> I,

coev: I -> E tensor E*,

where I is the monoidal unit.

The evaluation map describes annihilation into a neutral sector.

The coevaluation map describes creation of a particle-antiparticle pair from a neutral process, subject to conservation laws.

The duality conditions require that composing creation and annihilation in the appropriate order returns the original object up to coherent equivalence.

This expresses a deep relation between particle and antiparticle identity.

They are not unrelated objects carrying arbitrarily opposite labels. They are dual relational patterns whose transformation laws cancel under composition.

If E transforms under a representation rho, then E* transforms under the dual representation rho*.

For a phase charge q, the dual carries charge -q.

For non-Abelian symmetries, the dual representation reverses the relevant transformation behavior.

Antiparticles may also be interpreted as oppositely oriented defects. A particle and antiparticle carry inverse topological charges:

Q(E*) = -Q(E).

When brought together, their combined defect class may become trivial:

Q(E tensor E*) = 0.

Annihilation is then the removal of a paired coherence obstruction, with the released energy appearing as other allowed excitations.

Pair creation is the inverse process in which a neutral field configuration reorganizes into two oppositely charged defects while preserving total invariants.

Duality therefore gives antiparticles a structural origin within the compositional architecture of the theory.

7.9 Composite Matter

Physical objects combine to form larger systems. A relational theory must therefore explain composition without assuming that wholes are merely collections of independent parts.

Let A and B be physical identity patterns. Their composite is represented schematically by

A tensor B.

The tensor product does not necessarily mean simple juxtaposition. It specifies how the relational structures of A and B coexist and how their processes combine.

If the systems are independent, morphisms may factor:

Hom(A tensor B, A' tensor B')
~= Hom(A,A') tensor Hom(B,B').

When interaction or entanglement is present, this factorization can fail.

The composite may possess automorphisms, observables, and defects not reducible to those of the separate components.

A bound state is a composite equivalence pattern whose total coherence cost is lower than that of the separated components under the same conserved quantities.

Let

M(A),

M(B),

and

M(A tensor B)

denote the corresponding maintenance energies.

Binding occurs when

M(A tensor B) < M(A) + M(B).

The difference

E_bind =
M(A) + M(B) - M(A tensor B)

is released as energy or transferred to other degrees of freedom.

Composite matter may be formed through several categorical constructions.

Tensor products describe joint systems.

Products describe systems satisfying simultaneous projection properties.

Coproducts describe alternative or branching constructions.

Limits encode globally consistent solutions to families of local constraints.

Colimits encode objects assembled by gluing components along shared boundaries.

Quotients identify internal redundancies.

Extensions describe composites in which one structure is embedded nontrivially within another.

The correct construction depends on the physical form of composition.

This language is especially useful because it distinguishes a mere collection from an integrated whole.

A composite object exists as a stable entity when the relations among its components form a new persistent equivalence pattern.

Its identity is not simply the list of its parts.

It is the coherence of their composition.

7.10 Interaction as Coherence Exchange

An interaction changes the relational organization of physical identity patterns.

Suppose two objects A and B approach one another. Before interaction, their joint structure may approximately factor as

A tensor B.

During interaction, new morphisms connect their internal structures. Their equivalence classes, automorphism responses, transport costs, or defect configurations may become coupled.

After interaction, the result may be

A' tensor B',

a bound composite C,

a set of new excitations,

or a reconfigured relational network.

Interaction can therefore be described as coherence exchange.

The systems exchange not only energy and momentum, but constraints on how their identities can be transported and composed.

A force appears when the presence of one coherence pattern changes the transport cost of another.

If B alters the local equivalence geometry around A, the minimum-cost path of A changes. In a continuum limit, this appears as acceleration.

A mediator field can be interpreted as a propagating modification of the equivalence transport rules.

A gauge boson carries changes in internal comparison.

A gravitational excitation carries changes in spacetime comparison.

A scalar excitation carries changes in local coherence cost.

The interaction vertex in a conventional field theory becomes a compositional rule specifying which relational patterns may be joined.

Conservation laws constrain these rules. The composite morphism must respect global automorphism representations, topological charges, causal order, and boundary conditions.

An allowed interaction has the schematic form

f: A tensor B -> C tensor D

when the relevant invariants match across the transformation.

A forbidden interaction is one for which no admissible morphism exists.

Interaction strength may be related to the measure, multiplicity, or cost of the allowed morphism space

Hom(A tensor B, C tensor D).

A large space of low-cost transformations would produce a strong interaction.

A sparse or high-cost morphism space would produce a weak interaction.

A completely empty morphism space would forbid the process.

This offers a possible relational origin for coupling constants. A coupling could quantify the accessibility of coherence-changing transformations between specified identity sectors.

The proposal must eventually calculate such constants rather than merely interpret them.

The matter ontology developed in this section can be summarized as follows.

An object is a persistent relational pattern.

A particle is a stable local defect or irreducible coherence structure.

Mass is the cost of maintaining and transporting that structure.

Rest energy is its minimum internal coherence cost.

A massless excitation is a pattern existing only through propagation.

Charge is transformation under automorphisms.

Internal symmetry is self-equivalence of local identity.

An antiparticle is a dual or inverse coherence pattern.

A composite object is a new identity formed through coherent composition.

An interaction is a redistribution of equivalence, obstruction, and transport cost.

The conventional question asks:

What substance constitutes matter?

Univalent Relational Dynamics asks:

Which relational patterns remain coherent, which obstructions protect them, how do they transform under self-equivalence, and what does it cost to preserve them through change?

Under this view, matter is not an additional ingredient placed into geometry.

Matter is what stable coherence looks like when equivalence cannot be globally trivialized.

8. Quantum Theory as an Integral Over Equivalences

8.1 From State Vectors to Relational Amplitudes

Quantum mechanics is commonly formulated by assigning a state vector, ray, density operator, or algebraic state to a physical system. The state evolves according to a dynamical law, and measurement associates probabilities with possible outcomes. This formalism is empirically successful, but it leaves open the ontological status of the quantum state. Is it a physical object, a bookkeeping device, a catalogue of knowledge, or a representation of deeper relational structure?

Univalent Relational Dynamics proposes that the quantum state is a presentation rather than the fundamental entity. Its physical content lies in the network of transformations connecting preparations, processes, and observations.

Let x and y denote physical presentations. A quantum amplitude is not interpreted primarily as a property of x or y in isolation. It is assigned to the structured ways in which x can be related to y.

Write

A(x -> y)

for the amplitude associated with a transformation from x to y.

If several admissible transformations connect the same endpoints,

e_1, e_2, ..., e_n: x -> y,

then the total amplitude is obtained by combining the contributions of those transformations:

A(x -> y) = sum_k A(e_k).

The probability associated with the transition is then determined by the magnitude of the total amplitude:

P(x -> y) = |A(x -> y)|^2,

after appropriate normalization.

The decisive feature is that amplitudes add before probabilities are formed. Distinct transformations are therefore not merely alternative classical routes. Their complex contributions can reinforce or cancel one another.

In an equivalence-first interpretation, the quantum object is not a point moving through a state space. It is the coherent totality of admissible relations between preparation and outcome.

The state vector can still be used, but it becomes a compressed representation of these relational amplitudes. A ket |psi> summarizes how one preparation context is related to a family of possible observational contexts. It does not need to be regarded as a material wave existing independently in an abstract Hilbert space.

A basis change then becomes a change of presentation. The physical content lies in the invariant amplitude structure, not in the coordinates used to express it.

This view is consistent with the observation that absolute phases, basis labels, and many components of a quantum state are presentation-dependent. Only relational quantities, transition amplitudes, interference patterns, expectation values, and invariant correlations are operationally accessible.

The fundamental quantum question is therefore not

What state does the system possess?

It is

What coherent transformations connect the preparation, interaction, and observation contexts?

8.2 The Equivalence Path Integral

The path integral provides a natural starting point for equivalence-first quantum theory because it assigns amplitudes to entire histories rather than to isolated intermediate states.

In conventional notation, the transition amplitude between configurations x and y is written schematically as

K(x,y) = integral over paths gamma:x->y of exp(iS[gamma]/hbar) Dgamma.

Univalent Relational Dynamics generalizes this expression by integrating over admissible transformations and higher equivalence data:

K([x],[y]) =
integral over e:x->y of exp(iS[e]/hbar) De.

The brackets [x] and [y] indicate physical equivalence classes or higher moduli objects rather than arbitrary presentations.

The integration variable e may include:

a trajectory;

a gauge transport;

a field history;

a deformation of relational structure;

a change in topology;

a higher-categorical process;

or a complete coherence assignment on an interpolating complex.

The action S[e] measures the coherence cost, curvature, obstruction, and local maintenance associated with that transformation.

The path integral is therefore reinterpreted as an equivalence integral.

Each admissible way of carrying physical identity from one context to another contributes a phase

exp(iS[e]/hbar).

Transformations whose actions differ by much more than hbar tend to interfere destructively. Families near stationary action reinforce one another. Classical behavior emerges when the integral is dominated by a narrow set of coherent transformations.

The stationary condition is

delta S[e] = 0.

In conventional language, this selects a classical trajectory. In equivalence-first language, it selects a transport structure whose coherence cost is stationary under small deformation.

The integration domain must be defined carefully. If multiple presentations describe the same transformation, naive integration over all of them overcounts physical possibilities.

The correct integral should therefore be taken over a homotopy quotient or moduli structure:

Transformations / physical equivalence.

Symbolically,

K([x],[y]) =
integral over Mor(x,y)//E of exp(iS[e]/hbar) Dmu(e).

Here Mor(x,y) is the space of processes from x to y, E is the equivalence structure among those processes, and Dmu is a measure respecting the quotient.

Automorphisms must also be included. A transformation possessing a large stabilizer should not necessarily be weighted in the same way as one with trivial symmetry.

For a finite groupoid, a schematic sum would be

K([x],[y]) =
sum over [e] of exp(iS[e]/hbar) / |Aut(e)|.

This expression weights each transformation class by the inverse size of its automorphism group.

In a continuous theory, one must replace the finite group size with an appropriate group volume, determinant, or gauge-fixed measure.

The equivalence path integral therefore unifies several familiar procedures:

summing over histories;

quotienting gauge redundancy;

weighting symmetry sectors;

including topological classes;

and tracking higher coherence.

The mathematical challenge is to define such integrals rigorously for higher groupoids and infinity-categories. The physical challenge is to show that the resulting amplitudes reproduce known quantum predictions while providing additional structure or constraints.

8.3 Interference Among Identifications

Interference is often described as interference among paths. The equivalence-first interpretation is more general:

Interference occurs among distinct coherent ways of identifying preparation with outcome.

Consider two transformations

e_1, e_2: x -> y.

Their amplitudes are

A_1 = a_1 exp(i phi_1),

A_2 = a_2 exp(i phi_2).

The total probability is

P = |A_1 + A_2|^2.

Expanding gives

P = a_1^2 + a_2^2 + 2a_1a_2 cos(phi_1 - phi_2).

The final term is the interference term. It depends on the relative phase between the two identifications.

The physical significance of the relative phase is relational. Neither phi_1 nor phi_2 alone is generally observable. What matters is their difference.

This supports the broader claim that quantum theory does not assign absolute physical meaning to isolated amplitudes. It assigns meaning to relationships among alternatives.

A double-slit experiment can therefore be described without imagining a classical particle secretly choosing one path. The preparation and detection contexts are connected by a structured space of transformations. When the alternatives remain coherently comparable, their amplitudes combine.

If the environment records which alternative occurred, the transformations become correlated with distinguishable environmental presentations. Their equivalence space changes, and the cross terms become inaccessible or suppressed.

The essential distinction is not simply whether two spatial paths exist. It is whether the transformations remain members of one coherent comparison structure.

Interference may therefore occur among:

spatial trajectories;

internal symmetry transports;

topological sectors;

particle-number histories;

field configurations;

causal orders;

or higher categorical processes.

Aharonov-type phases, geometric phases, and topological interference can all be interpreted as residues of nontrivial identity transport.

The phase accumulated along e may be written schematically as

phi[e] = S[e]/hbar.

A loop in transformation space can produce a phase even when local force-like quantities vanish. This shows that the topology and connection of the equivalence space can affect observable outcomes.

Interference is thus evidence that alternative identifications are not merely separate possibilities. They belong to one higher relational object until some physical process destroys or hides their comparability.

8.4 Superposition as Unresolved Equivalence Structure

A quantum superposition is usually written as

|psi> = sum_i c_i |i>.

The basis states |i> represent possible outcomes or components, and c_i are complex amplitudes.

In a state-first ontology, this expression can appear to assert that the system is literally in several mutually exclusive states at once.

The equivalence-first interpretation is different. A superposition represents an unresolved higher relation among multiple presentations.

The alternatives |i> are not independently actual classical objects. They are boundary presentations within a coherent structure of possible transformations.

The coefficients c_i specify how the preparation context is related to each alternative.

The state |psi> therefore encodes a relational map

preparation -> family of outcome presentations.

Superposition persists as long as the alternatives remain coherently comparable. This means that phase relations among the c_i remain physically accessible.

A global phase transformation

|psi> -> exp(i theta)|psi>

does not change the physical ray. It alters the presentation while preserving all relative phases.

This is a simple example of univalent identity in quantum theory. Distinct vectors represent one physical quantum identity because an admissible phase equivalence connects them.

More generally, different decompositions of |psi> may represent the same abstract state:

|psi> = sum_i c_i |i> = sum_j d_j |j'>.

The components depend on the selected basis. The underlying physical content does not consist of one privileged list of alternatives.

Superposition should therefore not be identified with one basis-dependent decomposition. It is the capacity of the preparation to maintain coherent relations across multiple possible observational decompositions.

A measurement context selects a particular family of distinctions. The quantum structure is then projected or truncated relative to that context.

This interpretation avoids assigning primitive reality to every term in an arbitrary expansion. It places physical emphasis on the full relational structure and its response to observational decomposition.

Superposition becomes:

the continued availability of several mutually coherent identification paths.

Classical exclusivity appears when those paths can no longer be jointly compared.

8.5 Entanglement as Nonfactorizable Identity

Entanglement is one of the clearest cases in which the identity of a whole cannot be reduced to independent identities of its parts.

Let A and B be two subsystems. If their joint state factors as

|Psi_AB> = |psi_A> tensor |phi_B>,

then the systems possess independent pure-state presentations.

An entangled state cannot be written in this form.

For example,

|Psi_AB> =
(1/sqrt(2))(
|0>_A tensor |1>_B
+
|1>_A tensor |0>_B
).

The whole has a definite relational structure, while neither part possesses an independent pure state.

Univalent Relational Dynamics interprets this as nonfactorizable identity.

For separable systems, one expects a factorization such as

Equiv(A tensor B, A' tensor B')
~=
Equiv(A,A') x Equiv(B,B').

For entangled systems, this factorization fails:

Equiv(A tensor B, A' tensor B')
!=
Equiv(A,A') x Equiv(B,B').

The joint system admits transformations, invariants, and correlations that cannot be decomposed into independent structures belonging to A and B.

Entanglement therefore does not require a signal traveling between two fully independent objects at measurement. The deeper issue is that the objects were never completely independent at the level of physical identity.

Their apparent separation is a feature of a chosen presentation. Their joint equivalence structure remains nonlocal in the sense that it cannot be reconstructed from local factors alone.

This does not permit faster-than-light communication. Local outcome statistics remain constrained. The nonfactorizable structure appears in correlations revealed only when records are compared.

The relational interpretation distinguishes two concepts:

causal influence;

and shared identity structure.

Entanglement concerns the second.

A measurement on A changes the conditional presentation used to describe B, but it need not send a controllable dynamical signal to B. The joint object already contains the correlation structure.

Entanglement entropy measures the degree to which the identity of the whole fails to factor.

For a reduced density operator rho_A, the von Neumann entropy is

S_A = -Tr(rho_A log rho_A).

If S_A = 0 for a pure joint state, A and B are separable.

If S_A > 0, the subsystems participate in a nonfactorizable identity.

This suggests a geometric interpretation. Entanglement entropy measures the obstruction to decomposing one relational object into independent subobjects.

At a deeper level, spacetime connectivity itself may be related to patterns of entanglement. If so, nonfactorizable quantum identity would not occur inside a preexisting geometry. It could help generate the geometry that makes subsystem separation meaningful.

8.6 Contextuality

Quantum contextuality means that measurement outcomes cannot generally be understood as revealing a set of preexisting values assigned consistently to all observables independent of measurement context.

This phenomenon fits naturally within an equivalence-first framework.

A measurement context is a compatible family of observables that can be jointly evaluated. Different contexts may overlap, sharing some observables while differing in others.

A classical hidden-value model would assign global values v(O) to all observables such that every local context receives a consistent restriction.

Quantum theory obstructs such a global assignment in sufficiently rich systems.

In relational language, local observational presentations exist, but they cannot always be glued into one global classical state.

Let {C_i} be a family of measurement contexts. Each context admits local outcome assignments

v_i: C_i -> outcomes.

On overlaps C_i intersect C_j, the assignments may agree locally. Yet the entire family may fail to define one global assignment v.

The failure is a local-to-global coherence obstruction.

Contextuality therefore resembles a nontrivial gluing problem.

Each measurement context provides a valid local presentation.

No single context-free classical presentation reproduces all of them simultaneously.

The obstruction is not ignorance about hidden values. It is the absence of a globally coherent assignment compatible with the full relational structure.

This supports the presentation axiom. Quantum properties are not necessarily values possessed independently of the contexts in which they are defined.

It does not follow that outcomes are subjective or arbitrary. The relations among contexts are objective and experimentally constrained.

A contextual theory can produce highly precise statistical predictions while denying that all observables possess simultaneous context-independent values.

In Univalent Relational Dynamics, contextuality is expected whenever local truncations of a higher object cannot be combined into one set-like global presentation.

The underlying relational object may be coherent, while every attempt to flatten it into a classical global value table fails.

Contextuality then becomes another manifestation of higher structure resisting complete truncation.

8.7 Measurement as Truncation

Measurement converts a quantum description containing coherent alternatives into a classical record with a definite outcome.

Univalent Relational Dynamics models this transition as truncation.

Let R be a higher relational object containing presentations, paths, paths between paths, and associated amplitudes.

A classical outcome space retains distinguishable components but forgets higher identifications connecting them.

The simplest truncation is

tau_0: R -> pi_0(R).

The object pi_0(R) records the connected outcome sectors while discarding the internal path structure.

Before measurement, several alternatives may be connected through accessible coherence paths.

After measurement, the apparatus produces a stable record belonging to one classical sector. The higher paths linking that record to incompatible alternatives are no longer available within the observer’s effective description.

Measurement is therefore not merely the reading of a preexisting property. It is a physical process that changes the accessible level of relational structure.

A more detailed measurement model includes:

the system S;

the apparatus M;

the environment E;

and an interaction

U: S tensor M tensor E -> S tensor M tensor E.

Initially, the apparatus is in a ready presentation |M_0>. For a system superposition,

sum_i c_i |s_i>,

the interaction produces

sum_i c_i |s_i>|M_i>|E_i>.

The apparatus records |M_i> become correlated with system alternatives, and the environmental states |E_i> become effectively distinguishable.

The reduced presentation accessible to an observer no longer contains observable interference between different i values when the environmental overlaps satisfy

<E_i|E_j> ~= 0

for i != j.

The higher paths have not necessarily been deleted from a universal mathematical description. They have become inaccessible to the local observational context.

The effective truncation produces a classical mixture:

rho_eff ~= sum_i |c_i|^2 |s_i,M_i><s_i,M_i|.

This explains the suppression of interference but does not alone explain why one specific outcome is experienced.

The measurement problem therefore separates into at least two questions:

Why do higher equivalences become inaccessible?

Why is one classical record realized or experienced rather than the complete mixture?

The truncation framework directly addresses the first. The second requires an additional interpretation or dynamical law.

Possible approaches include:

objective stochastic selection;

branch-relative identity;

relational outcome actualization;

boundary-condition selection;

or a deeper rule governing observer-compatible truncation.

The framework should not claim to have solved the measurement problem until it derives both classical definiteness and quantum probabilities.

Its immediate contribution is to recast measurement as a change in accessible homotopy level.

8.8 Decoherence as Loss of Accessible Higher Paths

Decoherence occurs when a quantum system becomes entangled with environmental degrees of freedom in a way that suppresses observable interference among selected alternatives.

In Univalent Relational Dynamics, decoherence is the loss of accessible higher paths between presentations.

Suppose two alternatives e_1 and e_2 connect a preparation to an outcome. Their interference requires a comparison map preserving their relative phase.

Environmental interaction correlates them with different environmental presentations:

e_1 -> e_1 tensor E_1,

e_2 -> e_2 tensor E_2.

If E_1 and E_2 become effectively orthogonal, the comparison between the alternatives requires control over a vast environmental structure. For local observers, the necessary higher equivalence is no longer operationally available.

The alternatives then behave as separate classical sectors.

Decoherence does not necessarily destroy the total relational structure. It redistributes coherence into correlations with the environment.

Local coherence decreases while global entanglement increases.

This distinction is important. Information is not simply erased. It becomes nonlocal and practically unrecoverable.

The process can be expressed through the reduced density matrix. If

rho_SE = |Psi_SE><Psi_SE|,

then the local state is

rho_S = Tr_E(rho_SE).

Off-diagonal terms are multiplied by environmental overlaps. When

<E_j|E_i> -> 0,

those terms vanish from local predictions.

The pointer basis emerges from presentations that remain stable under environmental monitoring. These are the structures whose identity can be repeatedly copied into records without rapid disruption.

In equivalence-first language, pointer states are robust low-level presentations selected by the dynamics of truncation.

They possess high stability under the relevant system-environment morphisms.

Decoherence therefore helps explain why certain classical variables appear definite while arbitrary quantum superpositions do not.

Position-like, energy-like, or other stable observables emerge because their records are redundantly propagated through the environment.

Classical objectivity arises when many observers can access equivalent records without needing access to the suppressed higher paths.

The classical world is thus not a separate ontology added to quantum reality. It is a stable, redundantly recorded truncation of a deeper relational structure.

8.9 The Born Rule Problem

Quantum theory predicts probabilities through the Born rule:

P(i) = |c_i|^2.

Any proposed foundation of quantum mechanics must explain why squared amplitude, rather than amplitude magnitude, some other power, or a simple count of alternatives, determines observed frequency.

Univalent Relational Dynamics cannot treat the Born rule as an incidental detail. It must derive or reconstruct it from the geometry and measure of equivalence spaces.

Several possible routes exist.

The first route is norm preservation. If quantum transformations preserve an inner product, then the squared norm is the natural additive probability measure over orthogonal alternatives.

For

$|\psi\rangle = \sum_i c_i |i\rangle,$

normalization requires

$\sum_i |c_i|^2 = 1.$

The theory would need to explain why admissible equivalences carry a complex Hilbert-space structure and why physical probabilities must be represented by this norm.

The second route is measure on transformation space. The coefficient c_i may arise from a sum or integral over equivalences leading to outcome i:

$c_i =$
integral over e:x->i of exp(iS[e]/hbar) Dmu(e).

The probability $|c_i|^2$ then compares each transformation with a conjugate transformation:

$|c_i|^2 = c_i c_i^*.$

This can be interpreted as a measure on paired histories or equivalence loops formed by composing a process with its reverse.

The third route is symmetry. If outcome weights must remain invariant under unitary transformations, compose consistently for independent systems, and add over mutually exclusive alternatives, the squared norm may be uniquely selected.

The fourth route is decision-theoretic or frequency-based. Rational weighting or repeated composition may force branch weights to behave as $|c_i|^2$. Such arguments must be examined carefully because they may import assumptions equivalent to the rule they seek to derive.

The fifth route is homotopy cardinality or categorical trace. Probabilities may emerge from a measure on equivalence classes corrected by automorphism structure.

A schematic expression might be

$P(i) =$
$N_i / \sum_j N_j,$

where N_i is not a simple count but a weighted volume of the transformation groupoid leading to i:

N_i =
integral over Mor(x,i)//E dmu(e).

To reproduce quantum theory, this measure must include phase interference before yielding positive probabilities.

The Born rule problem therefore requires the theory to connect three levels:

complex amplitudes on transformations;

composition and interference of those amplitudes;

positive normalized probabilities on truncated outcomes.

A successful derivation would be one of the strongest results of the framework.

Until such a derivation is obtained, the theory should be regarded as an ontological and mathematical reformulation rather than a completed quantum foundation.

8.10 Quantum Field Theory

Quantum field theory combines quantum principles with locality, relativistic causality, particle creation, annihilation, and fields possessing infinitely many degrees of freedom.

An equivalence-first formulation should not begin with particles as permanent objects. It should begin with local relational structures and their compositional transformations.

Associate to each spacetime region U a structure A(U), such as an algebra of observables, category of boundary conditions, or space of local presentations.

If U is contained in V, there is a corresponding embedding or restriction relation between A(U) and A(V).

Locality requires that observables associated with appropriately separated regions satisfy compatibility or commutation conditions.

The assignment

U -> A(U)

should satisfy gluing rules so that global physics can be reconstructed from compatible local data.

In categorical language, quantum field theory becomes a functor from a category of geometric regions or bordisms to a category of algebraic or Hilbert-space structures.

A spacetime region acts as a process from its incoming boundary to its outgoing boundary.

The field theory assigns an amplitude, operator, or state-space map to that process.

Composition of regions corresponds to composition of physical processes.

This structure is already close to the requirements of Univalent Relational Dynamics. The new proposal is to treat both the geometric domain and the field presentations as emergent from a deeper equivalence category.

A field configuration phi is a local presentation.

Gauge transformations are equivalences among field presentations.

Particles are stable excitation sectors, defects, or irreducible representations of the local relational algebra.

Vacua are low-cost global coherence structures.

Creation and annihilation are morphisms changing the factorization and defect content of those structures.

Scattering amplitudes describe the transformation space between asymptotic equivalence patterns:

A_in -> A_out.

The perturbative expansion can be interpreted as a sum over compositional diagrams. Each Feynman diagram represents one presentation of a process, while physically equivalent diagrams, relabelings, gauge redundancies, and automorphisms must be correctly weighted.

The path integral is then

Z =
integral over fields/equivalence
exp(iS[phi]/hbar) Dphi.

A more complete expression would integrate over local presentations, transport maps, topology, and higher coherence:

Z =
sum over relational complexes K
integral Dphi DT Dh
exp(iS[K,phi,T,h]/hbar).

Here T denotes identity-transport data and h denotes higher coherence data.

Renormalization can also be given an equivalence-first interpretation.

Different microscopic presentations may flow toward the same effective macroscopic theory. A renormalization-group fixed point represents an equivalence class of descriptions stable under changes of scale.

Universality occurs when many distinct microscopic systems produce the same large-scale relational structure.

The renormalization flow is therefore a transformation in theory space, and physical phases are equivalence sectors under coarse-graining.

An effective field theory is not merely an approximation caused by ignorance. It is a valid presentation at a particular resolution, related to deeper presentations by structured scale transformations.

The major challenge is to recover the quantitative machinery of quantum field theory:

Lorentz invariance;

locality;

unitarity;

spin-statistics relations;

renormalization;

vacuum structure;

scattering amplitudes;

anomalies;

and the observed particle spectrum.

The equivalence-first framework must also explain anomalies as failures of classical equivalence to survive quantization. A symmetry present in the classical presentation may be obstructed in the quantum measure.

Such an anomaly is naturally interpreted as a higher-coherence failure.

The local transformations appear admissible, but no globally consistent quantum weighting respects all of them simultaneously.

Anomalies would therefore provide an important testing ground for the theory because they reveal where equivalence, measure, and global coherence interact nontrivially.

The quantum theory developed in this section can be summarized as follows.

Quantum states are presentations of relational amplitude structure.

Amplitudes are assigned to transformations rather than isolated states.

The path integral becomes an integral over equivalence classes of processes.

Interference occurs among coherent identifications.

Superposition is unresolved higher relation among alternatives.

Entanglement is failure of physical identity to factor.

Contextuality is failure of local classical presentations to glue globally.

Measurement is truncation of higher equivalence structure.

Decoherence is loss of local access to paths connecting alternatives.

Probability requires a measure on equivalence space capable of deriving the Born rule.

Quantum field theory becomes the local and compositional organization of these relational amplitudes across emergent geometry.

The conventional quantum question is:

Which state is the system in?

The equivalence-first question is:

What is the full space of coherent transformations connecting the physical contexts, how do those transformations interfere, and what remains when observation truncates their higher relations?

9. Time, Causality, Irreversibility, and Entropy

9.1 Reversible Equivalence and Irreversible Process

Univalent Relational Dynamics distinguishes two fundamentally different kinds of transformation. The first preserves physical identity and is reversible. The second produces genuine change and may not possess an inverse.

A reversible equivalence has the form

e: x -> y

together with an inverse

e^-1: y -> x,

such that

e^-1 o e ~= id_x

and

e o e^-1 ~= id_y.

The presentations x and y may differ, but the transformation preserves the structure required for them to count as the same physical reality. Coordinate transformations, changes of basis, gauge transformations, reversible transport, and symmetry operations provide familiar examples.

An irreversible process has the form

f: x -> y

without an admissible inverse returning y to x. The absence of an inverse may arise because information is discarded, correlations are dispersed, alternatives are selected, energy is dissipated, or a complex structure is replaced by a coarser one.

Examples include decay, measurement, thermalization, diffusion, coarse-graining, erasure, and the formation of stable records.

This distinction separates equivalence from dynamics.

Equivalence changes presentation while preserving physical identity.

Irreversible process changes the effective relational structure itself.

A physical theory containing only equivalences would describe a universe of reversible transformations but no genuine temporal production. Every process could be undone, and no persistent distinction between past and future would arise.

A physical theory containing only irreversible arrows would lack stable identity. Objects could change, but there would be no coherent criterion by which they could be recognized through those changes.

Reality therefore requires both:

reversible equivalences that preserve identity;

and noninvertible processes that generate history.

The interaction between them creates the possibility of time. Equivalences describe what remains the same. Noninvertible morphisms describe what becomes different.

9.2 Time as Directed Composition

In ordinary physics, time is introduced as a parameter t labeling states:

x(t).

Univalent Relational Dynamics instead begins with the compositional order of processes.

Suppose

f: x -> y

and

g: y -> z.

Their composite is

g o f: x -> z.

The order matters. In general,

g o f != f o g,

and the reverse composition may not even be defined.

This ordered composability provides a primitive temporal structure. The statement that f occurs before g means that the output boundary of f supplies the input boundary of g.

Time is therefore not initially a coordinate through which objects move. It is the ordering implied by the composition of noninvertible transformations.

A sequence

x0 -> x1 -> x2 -> x3

defines a process history. The intermediate presentations may later be assigned temporal coordinates, but their more fundamental relation is categorical order.

This permits time to exist even before a continuum clock variable is defined.

A clock is then a physical subsystem whose transformations form a sufficiently regular ordered sequence. Another process can be timed by comparing its composition with the clock sequence.

If the clock has presentations

c0 -> c1 -> c2 -> ...,

then the event x_i may be assigned a time by its relational correlation with some c_j.

Time becomes relational rather than absolute.

The theory must nevertheless explain why large-scale temporal order can be represented approximately by a real-valued parameter. This may occur when dense process composition admits a continuum approximation:

t in R.

The familiar differential equation

dx/dt = F(x)

would then be an effective description of a more primitive compositional process network.

The temporal coordinate summarizes ordering and rate. It does not create them.

9.3 The Universe as an Infinity-Category

A pure infinity-groupoid contains only invertible transformations. It describes the higher structure of equivalence but does not by itself represent irreversible evolution.

A universe containing both equivalence and genuine change is more naturally modeled by an infinity-category.

In such a structure:

objects represent local presentations;

1-morphisms represent physical processes;

some 1-morphisms are invertible equivalences;

other 1-morphisms are irreversible transformations;

higher morphisms relate alternative processes;

and higher coherence determines how those relations compose.

The invertible portion of the infinity-category forms an infinity-groupoid. This substructure describes physical sameness.

The noninvertible portion describes causation, preparation, decay, observation, coarse-graining, and other directed transformations.

The theory therefore separates two questions:

When do two presentations count as the same?

How can one physical situation produce another?

The first is answered by equivalence.

The second is answered by directed morphism.

The full category combines identity and history.

This structure also permits multiple histories between the same endpoints:

f, g: x -> y.

A higher morphism

alpha: f => g

may identify them as equivalent processes. Alternatively, no such higher morphism may exist, and the two histories may belong to distinct physical sectors.

Quantum amplitudes can then be associated with these alternative process morphisms. Classical history emerges when one family dominates or when higher comparisons become inaccessible.

The infinity-category is therefore not merely a bookkeeping device. It is a candidate ontology in which physical existence consists of composable processes and higher relations among them.

9.4 Causal Structure

Causality can be defined through the existence of directed physical processes.

Let

x <= y

mean that there exists an admissible causal morphism

f: x -> y.

The relation <= expresses that y can arise from, receive influence from, or contain a record of x.

For this relation to define a causal order, it should satisfy transitivity:

x <= y and y <= z imply x <= z.

This follows naturally from composition:

f: x -> y,

g: y -> z,

therefore

g o f: x -> z.

A suitable antisymmetry condition is

x <= y and y <= x imply x ~= y,

meaning that if two contexts are causally reachable from each other without generating a nontrivial cycle, they belong to the same causal equivalence class.

The causal future of x is

J+(x) = {y : x <= y}.

The causal past is

J-(x) = {y : y <= x}.

These sets can be defined before spacetime coordinates or light cones are introduced.

An emergent light cone appears as the boundary separating contexts reachable through admissible signal-like morphisms from those not reachable through such processes.

A maximal propagation speed arises if the relational rules restrict how quickly influence can spread through successive local compositions.

Causality therefore comes from constrained composability.

The theory must also address causal cycles. A loop of noninvertible processes

x -> y -> z -> x

could threaten ordinary temporal order. Some cycles may be reinterpreted as equivalence loops rather than genuine causal loops. Others may be forbidden by consistency conditions.

A possible causality principle is:

Every directed cycle of physically realizable processes must reduce to an admissible equivalence, or else carry a compensating obstruction that prevents logical inconsistency.

This principle would distinguish harmless reversible recurrence from paradoxical causal self-production.

The emergence of stable causal order may therefore require a category that is locally rich in morphisms but globally constrained against nontrivial irreversible cycles.

9.5 Entropy as Presentation Multiplicity

Statistical mechanics defines entropy through the multiplicity of microscopic states compatible with a macroscopic description.

If a macrostate m corresponds to Omega(m) microstates, then

$S(m) = k_B \log \Omega(m)$.

The equivalence-first interpretation replaces microstates with presentations and macrostates with coarse physical meanings.

Let

$q: P \to M$

map detailed presentations to macroscopic meanings. For m in M, the fiber

$q^{-1}(m)$

contains all detailed presentations that appear identical at the macroscopic level.

The equivalence entropy is then

$S_{eq}(m) = k_B \log \mathrm{Vol}(q^{-1}(m))$.

For a finite system,

$S_{eq}(m) = k_B \log |q^{-1}(m)|$.

This definition says that entropy measures how many distinguishable fine-grained presentations realize the same coarse meaning.

The entropy is therefore not simply disorder. A highly organized macrostate can possess high entropy if many microscopic presentations support it. Conversely, a visually irregular configuration can have low entropy if it is highly constrained.

The central quantity is multiplicity under coarse equivalence.

The definition also clarifies the observer-dependent aspect of entropy. Different observational resolutions induce different maps q and therefore different fibers.

A microscopic observer may distinguish presentations that a macroscopic observer treats as equivalent.

Entropy depends on which distinctions remain physically accessible.

This does not make entropy subjective. The observational context, resolution, and accessible algebra of measurements are themselves physical structures. Once they are specified, the equivalence classes and their multiplicities are objectively defined.

The thermodynamic state is therefore a truncated presentation of a deeper relational object.

9.6 Homotopy Cardinality

Simple counting can overestimate the number of physically distinct presentations when some presentations possess internal symmetries.

Suppose a finite groupoid G describes a collection of presentations and equivalences. Its homotopy cardinality can be defined schematically as

|G|_h = sum over [x] of 1 / |Aut(x)|.

The sum runs over equivalence classes [x], and Aut(x) is the automorphism group of a representative.

A presentation with a large automorphism group contributes less than a presentation with trivial symmetry because many formal descriptions are related by self-equivalence.

This correction is important in equivalence-first statistical mechanics. The relevant multiplicity is not the raw number of labeled configurations. It is the weighted size of the relational moduli structure.

A refined entropy may therefore be written

S_h = k_B log |G|_h.

For continuous groups, the finite quantity |Aut(x)| must be replaced by an appropriate group volume, determinant, or regularized measure.

Higher structures may require additional corrections from higher automorphisms. A full homotopy cardinality can contain alternating contributions from multiple homotopy levels.

Schematically,

weight(x) ~
|pi_2(x)| |pi_4(x)| ...
/
(|pi_1(x)| |pi_3(x)| ...),

when these groups are finite and the expression is well defined.

The precise formula depends on the mathematical setting, but the conceptual point is clear:

entropy should count physical presentations according to their equivalence structure, not merely their labels.

This may modify statistical weighting near highly symmetric states, gauge sectors, topological phases, and configurations with nontrivial stabilizers.

It also aligns statistical mechanics with path-integral methods, where gauge-equivalent configurations must not be counted as independent histories.

9.7 The Second Law

The second law states that the entropy of an isolated macroscopic system does not decrease under ordinary evolution:

Delta S >= 0.

In the equivalence-first framework, this becomes a statement about the growth of accessible presentation volume.

A system evolves from a macrostate m1 to a macrostate m2. If

Vol(q^-1(m2)) > Vol(q^-1(m1)),

then

S_eq(m2) > S_eq(m1).

High-entropy macrostates dominate because they can be realized by vastly more fine-grained presentations than low-entropy macrostates.

The system does not move toward disorder because disorder exerts a force. It moves toward relational sectors occupying greater measure in presentation space.

The second law can therefore be written as a typicality statement:

Under generic dynamics, a detailed presentation is overwhelmingly likely to enter coarse sectors with larger equivalence fibers.

This interpretation also explains why microscopic reversibility can coexist with macroscopic irreversibility.

The underlying transformations may be invertible, but the coarse-graining map q is many-to-one. A large number of microscopic histories produce the same macroscopic future, while only a tiny subset reconstructs a special low-entropy past.

The reverse evolution is not forbidden by microscopic law. It is statistically exceptional.

Entropy increase arises when dynamics spreads a presentation across a larger region of the fiber structure while observation retains only the coarse meaning.

The second law can thus be rephrased:

Closed systems tend toward macroscopic descriptions realized by larger spaces of mutually equivalent microscopic presentation.

This formulation links thermodynamics directly to the central ontology of the theory.

9.8 Information Loss and Truncation

Information is lost operationally when distinctions present at one descriptive level are absent at another.

Let R be a higher relational object and let

tau: R -> R_trunc

be a truncation or coarse-graining map.

Several distinct presentations in R may map to the same element of R_trunc:

tau(x1) = tau(x2) = ... = y.

The truncated observer cannot determine which x_i produced y.

The information loss is associated with the size and structure of the fiber

tau^-1(y).

A simple measure is

I_lost(y) = log |tau^-1(y)|.

A probabilistic form uses conditional entropy.

If X is the fine-grained variable and Y = tau(X) is the coarse variable, then the lost information is measured by

H(X|Y).

In a higher-categorical theory, truncation may forget not only objects but also paths and higher paths.

The map

tau_0: R -> pi_0(R)

forgets all internal equivalences within connected components.

The map

tau_1: R -> R_1

retains objects and ordinary paths but forgets higher coherence.

Each truncation level discards a different form of relational information.

This provides a common framework for several processes:

thermodynamic coarse-graining forgets microscopic distinctions;

decoherence hides phase relations among alternatives;

measurement produces set-like records from higher quantum structure;

renormalization forgets short-scale details;

classical geometry forgets microscopic relational fluctuations.

Information loss is therefore not always literal destruction. It can be loss of access to distinctions that remain encoded nonlocally or at a higher structural level.

The difference between fundamental loss and effective inaccessibility must be treated carefully.

If the full evolution is invertible, information may remain globally present while becoming inaccessible to local observers.

If the evolution itself is noninvertible, information may be genuinely removed from the effective theory.

Univalent Relational Dynamics must specify which type occurs in each physical setting.

9.9 Memory and Records

A physical record is a stable structure that correlates with a prior process.

Let x be an earlier context and r a later record. A record exists when there is a morphism

f: x -> r

such that some invariant feature of x is preserved in r.

The record need not reproduce x completely. It preserves enough relational information to distinguish among possible pasts.

A memory is therefore a persistent asymmetry in the equivalence network.

Before a record is formed, several histories may remain mutually comparable.

After record formation, one history becomes correlated with a stable physical structure. The record changes which transformations remain accessible.

A reliable record should satisfy at least four conditions.

It must be stable under ordinary environmental perturbation.

It must be reproducibly readable by multiple observers.

Its correlation with the recorded event must survive relevant changes of presentation.

Its formation must generate an asymmetry between the recorded past and the unrecorded future.

The asymmetry is crucial. We possess records of past events but not ordinary records of future events because record formation is a directed process.

A photograph, fossil, detector click, memory state, or written document is produced by a causal chain originating in an earlier event.

The reverse chain is not ordinarily available.

Record formation usually increases entropy because it disperses information into many degrees of freedom and creates redundant correlations.

A robust classical fact may be represented by many equivalent records distributed through the environment.

The more redundantly a fact is recorded, the more objective it appears. Multiple observers can access distinct physical copies and verify their equivalence.

Objectivity is therefore related to record redundancy.

A classical event becomes real for a community of observers when many stable presentation channels converge on the same truncated meaning.

Memory links time, entropy, and measurement. It converts transient process into persistent relational structure.

9.10 The Arrow of Time

The microscopic laws of many physical theories are approximately reversible, yet macroscopic experience exhibits a clear temporal direction.

Systems equilibrate.

Records accumulate toward what is called the future.

Heat flows from hotter bodies to colder ones.

Unstable structures decay.

Memories refer to the past.

The equivalence-first framework locates the arrow of time in the combined effects of noninvertible composition, low-entropy boundary conditions, coarse-graining, and growth of equivalence-fiber volume.

Several ingredients are required.

First, the universe must contain directed morphisms that are not merely reversible equivalences.

Second, the initial relational structure must occupy a restricted region of presentation space. A universe beginning near equilibrium would not produce a pronounced thermodynamic arrow.

Third, evolution must carry typical presentations toward coarse meanings with larger fibers.

Fourth, record formation must preserve traces of earlier low-entropy structure while dispersing correlations into the environment.

The temporal arrow can then be summarized as

restricted presentation volume
-> increasing relational multiplicity
-> stable records
-> apparent irreversible history.

This suggests that the arrow does not arise from time itself. It arises from how relational structures evolve along directed composition.

Time may provide order, while entropy provides orientation.

A bare sequence

x0 -> x1 -> x2

has an order but not necessarily a thermodynamic arrow.

The arrow appears when

$S_{eq}(x0) < S_{eq}(x1) < S_{eq}(x2)$

for typical coarse descriptions.

The direction of increasing entropy becomes the direction in which records proliferate and higher distinctions become inaccessible.

This interpretation also links the psychological arrow of time to physical record structure. An observer remembers one temporal direction because memory formation is itself an entropy-producing process embedded in that direction.

The framework must still confront the origin of the low-entropy initial condition. The second law explains why entropy tends to increase once such a condition exists. It does not explain why the early universe occupied such an exceptional relational sector.

Possible explanations include:

a boundary condition on the total relational history;

a dynamical selection principle favoring initially simple coherence structures;

a cosmological phase transition;

a constraint arising from global consistency;

or a two-boundary formulation in which temporal orientation emerges locally.

No explanation should be assumed without a concrete model.

The arrow-of-time problem therefore remains partly cosmological.

The concepts developed in this section form a connected hierarchy.

Equivalence supplies reversible identity.

Noninvertible morphisms supply genuine process.

Composition supplies temporal order.

Constrained composition supplies causality.

Coarse equivalence supplies macroscopic states.

Fiber volume supplies entropy.

Truncation supplies effective information loss.

Stable asymmetry supplies memory.

Entropy growth supplies temporal orientation.

The central conclusion is that time should not be treated merely as an external coordinate indexing states.

Time is the ordered structure of physical becoming.

Its arrow is the direction in which coherent distinctions are transformed into larger families of equivalent presentation, while records preserve partial traces of the transformations that produced them.

The conventional statement is:

A system changes as time passes.

The equivalence-first statement is:

Time emerges because irreversible processes compose, and its observable direction is defined by the growth, truncation, and recording of relational multiplicity.

10. The Genetic Code as a Finite Equivalence-First Toy Universe

10.1 Why Use a Nonphysical Finite Model

A proposed mathematical physics of equivalence requires examples simple enough to calculate completely. Fundamental physical systems are often infinite-dimensional, continuous, gauge-redundant, and technically difficult. Their equivalence classes may contain complicated stabilizers, singularities, and higher homotopy. A finite model cannot reproduce all of this structure, but it can isolate the central logical distinction between presentation and meaning.

The standard genetic code provides an unusually compact example. It contains a finite presentation space of sixty-four codons, a many-to-one map onto twenty amino acids and translation stop, a natural notion of local symbolic change, a graph structure, redundant coordinates, and an exact linear projection onto lower-dimensional variables.

The use of this model does not imply that biological translation is a fundamental physical theory. It does not imply that spacetime is literally constructed from codons or that elementary particles are biological symbols. The model is valuable because it displays, in a complete and reproducible system, several structures that an equivalence-first physics would need to formalize:

many presentations corresponding to fewer meanings;

local transformations that sometimes preserve meaning;

a quotient that retains nontrivial multiplicity;

a geometry whose ordering affects equivalence adjacency;

visible coordinates containing linear redundancy;

and a canonical reconstruction from a higher-dimensional field to lower-dimensional generators.

The genetic code is therefore used as a mathematical parable with exact combinatorial content. It shows how a finite symbolic universe can possess states, meanings, equivalence classes, local paths, self-loops, quotient topology, and extremal organization.

Its function is analogous to that of a cellular automaton, spin lattice, finite group action, or graph model in theoretical physics. The finite system is not claimed to be the universe. It is used to discover which concepts remain meaningful before taking a continuum limit.

10.2 Sixty-Four Symbolic Presentations

The RNA codon space consists of all ordered triples constructed from four symbols:

A = {G, A, C, U}.

A codon has the form

c = (c1,c2,c3),

where each coordinate belongs to A.

The number of possible codons is

|A|^3 = 4^3 = 64.

These sixty-four codons form a complete finite presentation space:

P_code = A x A x A.

Each codon is a distinct symbolic presentation. UUU, UUC, AUG, GCU, and UGG are different points in P_code because they differ in at least one coordinate.

At this stage, the codon space contains no equivalence relation and no amino-acid meaning. It is simply a product space of symbolic triples.

The three coordinates are ordered. A nucleotide in the first position is not equivalent to the same nucleotide in the second or third position. Thus the codons AUG and UAG are distinct despite containing the same three symbols.

This positional structure makes the codon space suitable for comparison with coordinate-based physics. A codon is specified by values assigned to three axes. The coordinates provide a complete local address, but they do not by themselves determine which distinctions will remain physically meaningful after translation.

The presentation space can be viewed in several ways.

As a set, it contains sixty-four elements.

As a Cartesian product, it contains three four-valued coordinates.

As a six-bit space, after assigning two binary variables to each nucleotide, it contains 2^6 addresses.

As a Hamming space, codons are separated by the number of coordinates in which they differ.

As an ordered cube, codons become vertices of a 4 x 4 x 4 lattice whose nearest-neighbor relations depend on the order chosen for the four symbols along each axis.

These descriptions are not identical. They expose different structures within the same presentation space. The set description records membership. The product description records coordinate factorization. The Hamming description records substitution distance. The ordered cube introduces a selected local topology.

This distinction is central to equivalence-first physics. A presentation space does not arrive with one inevitable geometry. Geometry depends on which transformations are treated as elementary and how the coordinate values are ordered.

10.3 Twenty-One Translated Meanings

The standard genetic code defines a map

phi: P_code -> M_code,

where M_code consists of twenty amino-acid labels and one translation-stop label X.

Thus,

|P_code| = 64,

while

|M_code| = 21.

The map phi is many-to-one. Several codons can have the same translated output.

For example, multiple codons encode leucine, multiple codons encode serine, and multiple codons encode arginine. Other amino acids are represented by smaller codon families. Methionine and tryptophan each have one standard codon.

The codon is the presentation. The translated label is the meaning.

Two codons c and d are synonymous when

phi(c) = phi(d).

This defines an equivalence relation:

c ~ d if and only if phi(c) = phi(d).

The equivalence classes are the inverse images

phi^-1(m)

for each meaning m in M_code.

The codon space is therefore partitioned into twenty-one fibers of unequal size.

The quotient set

P_code / ~

contains twenty-one elements. It records which translated meanings exist, but it forgets much of the internal structure of their codon families.

For each meaning, the quotient forgets:

how many codons encode it;

where those codons lie in presentation space;

whether they are adjacent;

whether they form one connected component or several;

how many elementary transformations preserve the meaning;

and how their internal paths differ.

This demonstrates why physical identity cannot always be represented adequately by a bare quotient set. The number of meanings alone does not describe the geometry of synonymy.

Two hypothetical codes could assign the same numbers of codons to the same numbers of meanings while arranging synonymous codons very differently. Their quotient sets would have the same cardinality, but their robustness, connectivity, and local equivalence structure would differ.

The genetic code therefore illustrates the difference between quotient meaning and quotient geometry.

10.4 Synonymy as Physical Equivalence

Within the finite analogy, synonymous codons represent distinct presentations with the same translated identity.

If

phi(c) = phi(d),

then c and d are equivalent with respect to translation.

This equivalence is typed. The codons are not identical in every possible sense. They may differ in nucleotide composition, mutation frequency, expression context, molecular interaction, or regulatory behavior. They are equivalent only under the selected interpretation map phi.

This qualification is important for the proposed physics. Physical equivalence is never an unrestricted declaration that two presentations are the same in all respects. It is defined relative to the structures preserved by the interpretation.

For the codon toy model, the preserved structure is translated output.

For a gauge theory, it may be gauge-invariant observables.

For spacetime geometry, it may be invariant intervals and causal relations.

For quantum theory, it may be transition probabilities and expectation values.

The genetic-code model therefore demonstrates that equivalence requires an explicit criterion.

Synonymy can be represented as a groupoid whose objects are codons and whose morphisms connect codons assigned to the same meaning. A minimal groupoid could include one abstract equivalence between every pair of synonymous codons. A richer groupoid could distinguish transformations by the number, order, or type of nucleotide substitutions required.

The second construction is physically more suggestive. It distinguishes direct local equivalence from equivalence reachable only through a sequence of changes.

If two synonymous codons differ by one elementary move, the equivalence is locally accessible.

If they differ by several moves, their common meaning may be globally identical but locally separated.

The identity class therefore possesses internal geometry.

This becomes a finite model of a general physical principle:

Two presentations may represent the same entity while differing in the accessibility, cost, and topology of the transformations connecting them.

10.5 The Codon Cube as Presentation Geometry

To assign a local geometry to P_code, order the four nucleotide symbols along each coordinate axis.

For an isotropic cube, the same order is used on all three axes. Let the order be

s1 - s2 - s3 - s4.

Each codon becomes a lattice vertex with coordinates determined by the positions of its three nucleotides in this ordered list.

Two codons are nearest neighbors when they differ in exactly one coordinate and the two nucleotide values in that coordinate are consecutive in the selected order.

For a 4 x 4 x 4 lattice, the number of nearest-neighbor edges is

3(4 - 1)4^2 = 144.

The factor 3 selects the axis along which the change occurs. The factor 4 - 1 counts adjacent steps along that axis. The factor 4^2 counts the coordinate choices on the other two axes.

This ordered-cube geometry is not the same as the complete Hamming graph.

In the complete single-substitution Hamming graph, any nucleotide can be replaced by any of the other three nucleotides at one position. Each codon has nine such neighbors, and the total number of undirected edges is

64 x 9 / 2 = 288.

The ordered cube retains only half of those edges. It declares some one-nucleotide substitutions elementary neighbors and others non-neighboring jumps.

The choice of symbol order therefore changes the local topology without changing the codons or their meanings.

This is the key reason the cube is relevant to emergent geometry. The underlying presentation set remains fixed. The interpretation map remains fixed. What changes is the rule of local adjacency.

A geometry can thus be evaluated according to how well it aligns with the equivalence classes already present in the system.

The genetic-code problem becomes:

Which ordering of the presentation coordinates places the greatest number of equal meanings next to one another?

This is a finite version of a much broader question:

Which geometry makes physical identity maximally local?

10.6 Self-Loops as Identity-Preserving Motion

Collapse all codons with the same translated label onto one vertex. The resulting quotient graph contains twenty-one vertices.

Every one of the 144 lattice edges is retained.

If an edge joins two codons with different translated meanings, it becomes an edge between two different quotient vertices.

If an edge joins synonymous codons, its endpoints collapse onto the same quotient vertex. It becomes a self-loop.

Let E_pi be the ordered-cube edge set associated with nucleotide order pi.

For an edge e = {c,d}, define the synonymous indicator

I_phi(e) =
1 if phi(c) = phi(d),
0 otherwise.

The number of self-loops is

L(pi) = sum over e in E_pi of I_phi(e).

Each self-loop represents an elementary movement through presentation space that preserves translated identity.

The codon changes.

The meaning does not.

This is a finite realization of the central statement:

Motion in presentation space need not be motion in meaning space.

A self-loop is therefore more than redundant coding. It is an identity-preserving path.

The loop count L(pi) measures how frequently elementary local change remains inside the same equivalence class.

A high value means that equivalent presentations are locally compact under the chosen geometry.

A low value means that equivalent presentations are more widely separated.

The same quotient meanings can therefore support different local equivalence densities depending on the presentation geometry.

This provides a finite analogy for gauge motion, symmetry motion, and topologically protected motion. In each case, a descriptive variable changes while a physical invariant remains fixed.

A self-loop also illustrates why quotienting can destroy information. If one retains only the twenty-one quotient vertices, all synonymous transitions disappear. The quotient set treats every self-loop as nothing more than identity.

The multigraph retains the multiplicity of those loops. It distinguishes an identity with many locally accessible presentations from an identity with only one presentation.

The internal pathways of sameness are therefore part of the structure.

10.7 The G-A-C-U Extremum

There are

4! = 24

possible isotropic nucleotide orders.

Each order produces the same number of codon vertices and lattice edges. Each uses the same genetic-code map. Only the coordinate order changes.

Exhaustive enumeration shows that the order

G-A-C-U

and its reversal

U-C-A-G

produce the maximum number of synonymous nearest-neighbor edges:

L_max = 44.

No other isotropic order reaches this value.

The two orders represent the same undirected axis path viewed in opposite directions. They therefore define one geometric solution up to reversal.

This result has a precise but limited interpretation.

It does not show that G-A-C-U is optimal under every imaginable measure.

It does not show that the genetic code was generated by this order.

It does not establish a cosmological or metaphysical message.

It shows that, among the twenty-four common nucleotide orders applied isotropically to the three codon positions, one undirected ordering uniquely maximizes local preservation of translated identity.

The presentation geometry is therefore not neutral with respect to meaning.

One coordinate order aligns more strongly than the others with the code's equivalence classes.

This permits a new reading of preferred geometry. A preferred geometry need not be selected because it looks symmetric in the presentation variables alone. It may be selected because it best localizes the invariant meanings carried by those variables.

The extremum suggests the principle:

A geometry becomes physically distinguished when it makes equivalence local.

In a fundamental theory, the analogous criterion might favor relational networks in which the transport of persistent identity requires fewer changes, lower action, or less coherence correction.

The G-A-C-U result is not evidence that this universal principle is correct. It is a concrete finite case in which the principle can be stated, calculated, and tested exhaustively.

10.8 An Equivalence Action for the Cube

The self-loop count can be converted into a simple action-like quantity.

Let

$|E_{pi}| = 144$

for every isotropic order pi.

Define the equivalence-loss action

$S_{eq}(pi) = |E_{pi}| - L(pi)$.

This counts the number of nearest-neighbor edges that change translated meaning.

For the maximizing order,

S_eq(G-A-C-U) = 144 - 44 = 100.

Since |E_pi| is constant, minimizing S_eq is exactly equivalent to maximizing L.

The selected geometry is therefore the geometry minimizing local loss of meaning.

This action is deliberately simple. Every identity-preserving edge receives the same reward, and every identity-changing edge receives the same cost.

A richer action could weight edges by the magnitude of the change they produce.

Let Q(m) be a property vector associated with meaning m. Define an edge cost

C(e) = ||Q(phi(c)) - Q(phi(d))||^2

for e = {c,d}.

Then define

S_property(pi) = sum over e in E_pi of C(e).

This functional favors geometries in which neighboring presentations either preserve meaning or change to physically similar meanings.

The binary loop action asks whether meaning changes.

The weighted property action asks how much meaning changes.

One could also include transport asymmetry, mutation likelihood, coordinate cost, or higher-order plaquette terms.

A generalized action might be

S(pi) =
alpha sum_edges [1 - I_phi(e)]

- beta sum_edges C_property(e)
- gamma sum_plaquettes K(plaquette)
- delta S_complexity(pi).

The first term penalizes loss of identity.

The second penalizes large property changes.

The third measures coherence around elementary squares.

The fourth penalizes excessive geometric complexity.

This construction mirrors the proposed relational action of Univalent Relational Dynamics. A geometry is selected by balancing local equivalence, transformation cost, loop coherence, and structural simplicity.

The finite model is especially valuable because every candidate geometry can be enumerated. There is no need to rely on approximation or asymptotic reasoning.

10.9 Rank Deficiency and Hidden Degrees of Freedom

Each codon can be represented by twelve binary nucleotide-position indicators.

There are four possible nucleotide indicators for each of three positions:

3 x 4 = 12.

Let D be the 64 x 12 codon matrix. Each row contains exactly three ones, one in each positional block.

Although D has twelve columns, its rank is ten.

The reduction occurs because the indicator columns satisfy positional sum constraints. Within each codon position, the four nucleotide indicators sum to one.

Let the four columns for position p be D_p,G, D_p,A, D_p,C, and D_p,U. Then

D_p,G + D_p,A + D_p,C + D_p,U = 1

for each p.

The three positional sums are all equal to the same all-ones vector. Thus only one of the three block-sum directions is independent, producing two linear dependencies.

The visible coordinate count is twelve.

The independent linear dimension is ten.

This provides a finite example of representational redundancy.

The twelve coordinates are useful and natural. None is individually meaningless. Yet the complete set contains more coordinates than the underlying additive function space requires.

The lesson generalizes:

A descriptive system may use more coordinates than there are independent physical degrees of freedom.

In physics, similar redundancy appears in constrained systems, gauge potentials, homogeneous coordinates, tensor components, and coordinate charts.

Rank deficiency is not necessarily an error. It may be the algebraic signature of a representation designed for symmetry or interpretability.

The important task is to identify the null directions. Changes along those directions alter the coordinate coefficients without changing the reconstructed codon field.

Such directions resemble gauge freedoms. They represent alternative parameter assignments producing the same observable output.

The genetic-code matrix therefore supplies a transparent finite case in which visible variables, latent rank, and nonunique coefficients can be calculated exactly.

It supports the broader warning:

Do not identify the number of labels in a presentation with the number of independent dimensions in reality.

10.10 The Pseudoinverse as Latent Reconstruction

Because the codon matrix D is rectangular and rank deficient, it does not possess an ordinary inverse.

Nevertheless, the Moore-Penrose pseudoinverse

D+

provides a canonical mapping from codon-level property fields back to nucleotide-position coefficients.

Let

y in R^64

assign a numerical value to each codon. The minimum-norm additive coefficient vector is

b = D+ y.

The reconstructed additive component is

y_hat = D b = D D+ y.

The operator

P = D D+

is the orthogonal projection onto the rank-10 column space of D.

Thus,

y = y_hat + r,

where

r = y - y_hat

is the residual component orthogonal to the additive nucleotide-position model.

This decomposition separates a high-dimensional field into two parts:

a component recoverable from lower-dimensional symbolic generators;

and a residual interaction component not expressible as a sum of independent coordinate contributions.

In the equivalence-first analogy, the pseudoinverse acts as a latent reconstruction map. It asks which lower-dimensional coordinate structure most consistently explains an observed field distributed across the full presentation space.

The reconstruction is not a complete inversion. Several codon fields can share the same additive projection. Information contained in the residual is lost if only b is retained.

The pseudoinverse therefore embodies a general principle of scientific inference:

A high-dimensional observable world may permit a canonical best reconstruction of lower-dimensional generators without being uniquely determined by them.

This resembles the inverse problem faced by physics. Observations are made in a large space of outcomes, while theory seeks a smaller set of latent causes.

The pseudoinverse is exact as a mathematical operator, but its reconstructed model may still be incomplete. Exact inference machinery does not imply that the chosen latent model captures all structure.

This distinction is crucial for any proposed fundamental physics. A compact relational grammar may explain a large fraction of observations while leaving residual terms corresponding to interactions, nonlinearities, context dependence, or hidden degrees of freedom.

10.11 Limits of the Analogy

The genetic-code model must be used with strict limits.

First, synonymous codons are not physically identical in every biological context. They can differ in expression rate, molecular recognition, mutation bias, and regulatory effects. Their equivalence is defined only with respect to translated amino-acid identity.

Second, the ordered codon cube is one selected geometry among many. Its nearest-neighbor edges depend on a chosen nucleotide order. The G-A-C-U maximum is therefore conditional on the isotropic ordered-cube construction and the self-loop statistic.

Third, the loop count measures one form of local compactness. It does not establish optimality under spectral geometry, total path length, energetic cost, error probability, graph curvature, or every possible physicochemical measure.

Fourth, the rank-10 codon matrix results from complete factorial one-hot encoding. Its rank deficiency is mathematically exact, but it should not be confused automatically with the dimensionality of spacetime or a fundamental physical gauge group.

Fifth, the pseudoinverse reconstructs additive coordinate contributions. It does not establish that all emergent reality is a linear projection.

Sixth, the many-to-one codon map is externally specified by the standard genetic code. A fundamental equivalence-first physics must explain where its interpretation map, admissible equivalences, and action arise. It cannot assume its own quotient structure without justification.

Seventh, a finite graph does not contain the analytical, causal, quantum, and relativistic structures required for physical theory. It has no automatic Lorentzian signature, Born rule, field dynamics, or gravitational limit.

The proper conclusion is therefore methodological rather than cosmological.

The model demonstrates that the following combination is mathematically possible:

a finite presentation space;

a many-to-one meaning map;

identity-preserving local transformations;

a nontrivial quotient multigraph;

an extremal coordinate geometry;

redundant visible coordinates;

and a canonical lower-dimensional reconstruction.

It does not show that the universe uses the same architecture.

10.12 The Transferable Principle

The most important result of the toy model is a transferable question.

Given:

a presentation space P;

a meaning map q: P -> M;

a family of candidate geometries G;

and a set of elementary transformations for each geometry;

which geometry makes equivalence most local?

For a candidate geometry g, define an identity-preservation functional

L(g) =
sum over elementary edges {x,y} in g of
I[q(x) = q(y)].

A preferred geometry may maximize L.

Equivalently, define an identity-loss action

S_loss(g) =
sum over elementary edges {x,y} in g of
[1 - I(q(x) = q(y))].

A preferred geometry may minimize S_loss.

For continuous or weighted systems, replace the binary indicator with a divergence between meanings:

S(g) =
integral over local pairs (x,y)
D_M(q(x),q(y)) dmu_g(x,y).

The selected geometry minimizes local change in physical meaning.

This produces a candidate general principle:

Physical geometry is an arrangement of presentations that extremizes the local preservation and coherent transport of identity.

The principle is stronger than ordinary smoothness. Smoothness asks whether nearby coordinates have similar values. Equivalence localization asks whether nearby presentations remain within the same physical identity structure or move between meanings at minimal cost.

The principle is also distinct from simple redundancy. Redundancy counts how many presentations represent one meaning. Equivalence localization asks how those presentations are arranged and connected.

A meaning represented by six isolated presentations may be highly redundant but locally fragile.

A meaning represented by six mutually adjacent presentations may be equally redundant but locally robust.

The geometry of equivalence therefore matters independently of class size.

This distinction may be relevant to physical models in which gauge-equivalent configurations, quantum histories, topological sectors, or coarse-grained states possess nontrivial internal organization.

The genetic-code example suggests a research strategy.

Construct finite presentation spaces.

Define explicit interpretation maps.

Enumerate candidate local geometries.

Measure identity-preserving adjacency.

Introduce weighted coherence actions.

Study quotient multigraphs and homotopy structure.

Determine whether extremal geometries exhibit stable dimension, locality, curvature, and defect formation.

Then investigate continuum limits.

The deepest lesson of the finite model can be expressed without biological language:

A world may contain many descriptions, fewer meanings, and a geometry selected by how efficiently those descriptions preserve meaning under local change.

In such a world, identity is not attached to one point.

Identity is distributed across a connected family of presentations.

A preferred coordinate order is not merely a convenient display.

It is the order that makes sameness locally navigable.

The codon toy universe therefore gives a finite and exact illustration of the paper's central thesis:

Reality need not be built from isolated states. It may be built from equivalence classes together with the paths, loops, costs, and geometries that allow different states to count as the same.

11. Mathematical Formalization and Computational Research Program

11.1 Selecting the Foundational Category

Univalent Relational Dynamics requires a mathematical setting capable of representing presentations, reversible equivalences, irreversible processes, higher relations among processes, local-to-global gluing, and dynamical change. No single elementary category is sufficient for every part of this task.

An ordinary category provides objects, morphisms, identity morphisms, and composition. It is adequate for representing processes and their sequential organization, but it does not automatically record transformations between processes.

A 2-category introduces 2-morphisms between 1-morphisms. This is sufficient for theories in which alternative processes can themselves be compared, deformed, or declared equivalent.

Higher categories extend this hierarchy. An infinity-category allows morphisms between morphisms at arbitrarily high levels. In an infinity-category of physical processes, basic morphisms may be irreversible, while higher morphisms are commonly treated as invertible equivalences.

An infinity-groupoid is appropriate for the reversible sector of the theory. It records presentations, equivalences between presentations, equivalences between equivalences, and all higher coherence data.

A complete formulation may therefore require at least two related structures:

an infinity-groupoid of physical identity;

and an infinity-category of physical process.

The infinity-groupoid answers the question:

Which presentations count as the same physical structure?

The infinity-category answers:

Which processes can transform one physical structure into another?

A monoidal structure is also required because physical systems combine. If A and B are systems, their joint system is represented by

A tensor B.

The monoidal product must be compatible with processes:

f: A -> A',

g: B -> B',

imply

f tensor g: A tensor B -> A' tensor B'.

A symmetric monoidal category is appropriate when exchanging independent systems does not alter the composite structure. A braided or non-symmetric monoidal category may be required when exchange carries nontrivial physical information.

A dagger structure may represent process reversal or adjoint operation:

f: A -> B

has

f_dagger: B -> A.

However, the dagger should not be interpreted automatically as a physically realizable inverse. A measurement or decay may possess a formal adjoint without possessing an accessible reverse process.

Higher topos theory offers another possible foundation. A higher topos can support spaces, local data, sheaves, homotopy types, logical propositions, and geometric constructions within one framework. It may therefore provide a setting in which spacetime, fields, observables, and identity types are represented internally rather than imposed externally.

Derived geometry may be required where moduli spaces contain singularities, constraints, or hidden infinitesimal structure. Ordinary quotient spaces often lose information at symmetric or degenerate configurations. Derived structures retain deformation and obstruction data.

The foundational choice should not be made solely by mathematical elegance. It must be judged by physical requirements:

Does it represent both reversible equivalence and irreversible process?

Does it support local-to-global reconstruction?

Does it admit composition of systems?

Can it represent probability and quantum amplitude?

Can it produce effective geometry?

Can it support computation?

Can it recover known physical theories?

The likely outcome is not one universal category used without modification. The theory may require a hierarchy of related mathematical environments, with functors connecting discrete, quantum, geometric, and observational descriptions.

11.2 Defining Admissible Physical Equivalence

The central undefined term in an equivalence-first theory is admissible.

Mathematics contains many notions of equivalence:

bijection;

homeomorphism;

diffeomorphism;

isometry;

unitary equivalence;

gauge equivalence;

homotopy equivalence;

categorical equivalence;

Morita equivalence;

and derived equivalence.

These relations preserve different structures. Two systems can be equivalent in one sense and inequivalent in another.

Univalent Relational Dynamics therefore cannot declare that all mathematical equivalences constitute physical identity. The theory must specify a physically relevant subspace

Equiv_phys(x,y)

within a larger space of formal transformations.

An admissible physical equivalence should preserve the structures regarded as observable or law-defining at the scale under consideration.

A candidate equivalence e: x -> y may be required to preserve:

transition probabilities;

causal relations;

conserved quantities;

interaction structure;

boundary conditions;

symmetry representations;

topological sectors;

composition laws;

or the values of a designated observable algebra.

One possible definition is operational:

e is physically admissible when no allowed experiment within the theory distinguishes x from e(x).

This criterion is attractive but incomplete. The class of allowed experiments may itself depend on the theory, scale, energy, or observational context.

A second definition is functorial. Let O be the family of physical observables. Then e is admissible when

O(e): O(x) ~= O(y)

for every O in the designated observable family.

This definition makes admissibility relative to an observational category.

A third definition is dynamical. A transformation is admissible when it preserves the action, measure, and composition laws:

S[e(x)] = S[x],

Dmu(e(x)) = Dmu(x),

and

e(g o f) ~= e(g) o e(f).

A fourth definition is structural. The transformation must be an equivalence within the selected physical category, preserving all data included in the object type.

These definitions may coincide in a complete theory, but they need not coincide in an effective theory.

Physical equivalence can also be scale-dependent. Two microscopic presentations may be distinct at high resolution but equivalent after coarse-graining. Let R_L denote a resolution map at scale L. Then one may define

x ~=_L y

when

R_L(x) ~= R_L(y).

The equivalence relation becomes weaker as resolution decreases because more distinctions are discarded.

This creates a hierarchy of physical identity:

exact microscopic equivalence;

effective low-energy equivalence;

thermodynamic equivalence;

classical observational equivalence.

A formal theory should record the maps connecting these levels rather than treating one as universally fundamental.

Admissibility must also be stable under composition. If

e: x ~= y

and

f: y ~= z

are admissible, then

f o e: x ~= z

must be admissible.

The identity transformation must be admissible, and admissible equivalences must possess admissible inverses.

Higher admissibility is also required. If two admissible transformations are physically equivalent, the higher map relating them must preserve the corresponding coherence structure.

The theory therefore needs not only a set of allowed transformations but a complete admissible higher subcategory.

11.3 Local Observables and Descent

Observables must be defined in a way that respects equivalence and locality.

Let C be the category of physical presentations and processes. An observable may be represented as a functor

O: C -> V,

where V is a category of values, probability distributions, vector spaces, operator algebras, or experimental records.

Functoriality requires

O(g o f) = O(g) o O(f)

and

O(id_x) = id_O(x).

Thus the observable respects composition of physical processes.

If e: x ~= y is an admissible equivalence, then O(e) must be an equivalence in V. A set-valued invariant may satisfy

O(x) = O(y).

A vector-space-valued observable may satisfy

O(x) ~= O(y)

through a specified linear isomorphism.

A quantum observable may transform covariantly under a change of basis while retaining identical expectation values.

Locality requires an assignment of observables to regions or contexts. Let U denote a local region. Associate an observable structure

A(U).

If U is contained in V, there is a restriction or inclusion map relating A(U) and A(V).

For a sheaf-like structure, observables defined on overlapping regions must agree on intersections and glue uniquely into a global observable when the compatibility conditions are satisfied.

For a cosheaf-like structure, local processes may combine into a global process through colimit-like assembly.

The distinction depends on whether the theory emphasizes restriction of information or composition of local systems.

Descent expresses the requirement that local data related by admissible equivalences define one global physical quantity.

Suppose local presentations x_i are defined on regions U_i, with overlap equivalences

e_ij: x_i ~= x_j.

An observable O descends if the local values satisfy

O(e_ij): O(x_i) ~= O(x_j)

and these equivalences obey coherence on triple and higher overlaps.

A failure of descent may indicate:

an anomaly;

a topological obstruction;

a nontrivial bundle;

contextuality;

or inconsistency in the proposed observable.

The observable-descent condition is therefore both a construction rule and a diagnostic.

A computational implementation should verify descent automatically. Given local data and transition maps, an algorithm should test:

pairwise consistency;

triple-overlap coherence;

higher cocycle conditions;

and invariance under change of presentation.

This would allow the theory to distinguish a genuine global physical observable from a collection of locally compatible-looking values that cannot coexist globally.

11.4 Coherence Functionals

The theory requires explicit numerical or algebraic measures of coherence.

A coherence functional should quantify how well a network of local presentations and transformations behaves as one persistent physical structure.

Consider a finite complex with vertices i, local presentations x_i, and transport maps T_ij on edges.

A local mismatch functional may be defined as

C_edge =
sum over edges (i,j)
d_X(T_ij(x_i),x_j)^2,

where d_X measures disagreement between the transported presentation and the target presentation.

A loop-coherence functional may be

C_loop =
sum over loops l
d_G(T_l,id)^2,

where T_l is the composite transport around l and d_G measures distance from the identity transformation.

A curvature-like plaquette functional may take the form

C_curv =
sum over elementary faces f
||log T_boundary(f)||^2.

A higher associativity functional may compare alternative compositions. For morphisms f, g, and h, define

C_assoc =
d_2(
h o (g o f),

(h o g) o f
)^2,

where d_2 is a measure on 2-morphisms or process equivalences.

A gluing functional may penalize disagreement on overlaps:

C_glue =
sum over overlaps U_i intersect U_j
d(
x_i restricted to overlap,
e_ij applied to x_j restricted to overlap
)^2.

A total coherence action may then be

S_coh =
alpha C_edge

- beta C_loop
- gamma C_assoc
- delta C_glue
- eta C_boundary.

The coefficients determine the relative physical cost of local mismatch, loop obstruction, higher incoherence, failed descent, and boundary structure.

The choice of norm or divergence is not arbitrary. It must be invariant under admissible changes of presentation.

If the theory uses group-valued transports, a bi-invariant group metric may be appropriate.

If the theory uses probability distributions, relative entropy or information distance may be appropriate.

If it uses density matrices, trace distance, fidelity, or quantum relative entropy may be relevant.

If it uses categories, one may require a categorical or homotopical notion of distance.

The action should also be local enough to permit composition and continuum limits.

One possible principle is that global coherence cost can be approximated by sums over cells:

S =
sum_vertices S_0

- sum_edges S_1
- sum_faces S_2
- sum_3-cells S_3
- ...

The n-cell term measures coherence at relational level n.

This hierarchy mirrors higher gauge theory, where connections may assign data to paths, surfaces, and higher-dimensional cells.

The physically realized structures may be selected by:

minimization of S_coh;

stationary action;

stochastic sampling weighted by exp(-S_coh);

or quantum amplitudes weighted by exp(iS_coh/hbar).

The same functional family could therefore support classical, statistical, and quantum formulations.

11.5 Discrete Equivalence Networks

The first computational models should be finite.

A discrete equivalence network consists of:

a finite set of vertices;

a presentation type assigned to each vertex;

directed or undirected edges representing processes or equivalences;

labels on edges specifying transport maps;

faces representing relations among edge compositions;

and higher cells representing higher coherence.

A graph records objects and 1-morphisms but cannot fully represent relations among paths. A simplicial complex or cubical complex adds triangles, squares, tetrahedra, cubes, and higher cells.

A simplicial model is useful when relations are based on arbitrary finite overlaps.

A cubical model is useful when transformations are coordinate-like or when independent directions should compose in square diagrams.

A directed acyclic graph is useful for causal process order.

A groupoid-valued graph is useful for reversible equivalence.

A higher-dimensional rewriting system is useful when processes transform into alternative processes.

A finite model should specify at least:

P, the presentation set;

E, the allowed elementary transformations;

Q, the interpretation or quotient map;

C, the coherence-cost rule;

and U, the update or variational rule.

The genetic-code toy system has exactly this form:

P is the codon set;

E is the selected nearest-neighbor relation;

Q is the translation map;

and C can count or weight changes in translated meaning.

A physical toy model should extend this architecture by making Q and E dynamical rather than externally fixed.

One research direction is adaptive equivalence networks. The edge weights change according to how consistently identity is transported. Frequently coherent paths become lower cost. Incoherent paths weaken or disappear.

A generic update could be

w_ij(t+1) =
w_ij(t)

- epsilon d(T_ij(x_i),x_j)
- reinforcement from coherent loops.

The network then learns a geometry from repeated successful identity transport.

Another direction is defect generation. Begin with locally trivial transport and impose boundary conditions that force nontrivial winding. Study whether stable localized structures emerge.

A third direction is dimensional emergence. Begin with a highly connected graph and minimize a coherence action containing penalties for nonlocal inconsistency. Measure whether the resulting graph develops lattice-like dimensional scaling.

A fourth direction is causal emergence. Allow only directed processes satisfying local composition rules. Test whether stable partial orders, causal cones, and finite propagation speeds appear.

Finite models are valuable because all states, paths, loops, and automorphisms may be enumerated for small systems. Exact calculation can precede approximation.

11.6 Continuum Reconstruction

A fundamental discrete theory must explain why macroscopic physics is accurately described by smooth manifolds, differential equations, and continuous fields.

Continuum reconstruction begins by defining a sequence of increasingly refined complexes

K_1, K_2, K_3, ...

with characteristic spacing a_n tending toward zero.

Each complex carries presentation data x_i and transport maps T_ij.

A continuum limit exists when large-scale observables converge as

a_n -> 0.

The local transport maps may be expanded as

T_ij = exp(a A_mu + O(a^2)),

where A_mu is an effective connection.

The loop transport around an elementary face becomes

T_loop =
exp(a^2 F_mu_nu + O(a^3)).

A discrete curvature action of the form

sum_faces [1 - Re Tr(T_loop)]

can approach

integral Tr(F_mu_nu F^mu_nu) dV

after suitable normalization.

Similarly, local presentation mismatch may yield

sum_edges ||x_j - T_ij x_i||^2

approaching

integral ||D_mu phi||^2 dV.

The continuum limit must also reconstruct topology and metric.

One method is spectral. Construct a graph Laplacian L_n and study the convergence of its low-lying eigenvalues and eigenvectors. If the spectrum approaches that of a differential operator on a manifold, an effective geometry can be inferred.

Another method uses diffusion. The scaling of random-walk return probability estimates spectral dimension.

Another uses neighborhood growth. If the number of nodes within relational distance r behaves as

N(r) ~ r^d,

then d is an effective dimension.

Another uses persistent homology to identify stable cycles across scales.

A successful continuum reconstruction should produce:

a stable effective dimension;

locality;

smooth transport;

finite curvature;

a consistent measure;

and approximate symmetries such as Lorentz invariance.

The continuum theory must not depend sensitively on microscopic labeling. Different fine networks should flow toward the same macroscopic equivalence class when they belong to one universality class.

Renormalization is therefore central. A coarse-graining map

R_b: K_n -> K_(n/b)

should preserve long-range relational observables while discarding fine presentation detail.

A fixed point of this transformation corresponds to scale-invariant relational structure.

Relevant perturbations survive at large scales.

Irrelevant perturbations disappear.

The continuum laws of physics may therefore describe the stable fixed-point behavior of broad classes of microscopic equivalence networks rather than one uniquely specified lattice.

11.7 Homotopy-Aware Computation

Standard graph algorithms track vertices, edges, shortest paths, connected components, and spectra. Univalent Relational Dynamics also requires the computation of loops, surfaces, higher coherence, automorphisms, and path equivalences.

Persistent homology can identify topological features that survive across scales. Given a filtration of a relational complex, it tracks the birth and death of connected components, loops, cavities, and higher-dimensional cycles.

The resulting persistence diagram can distinguish stable structural defects from short-lived combinatorial noise.

However, homology alone may be insufficient because it discards some non-Abelian and higher-homotopy information.

Homotopy-aware computation may require:

fundamental-group calculations;

higher homotopy approximations;

simplicial and cubical sets;

chain complexes;

cohomology with local coefficients;

higher-dimensional rewriting;

and computational category theory.

Path equivalence can be treated through rewriting rules. If two compositional expressions differ by an admissible coherence transformation, they belong to the same process class.

A rewriting engine should test whether two paths

p = f_n o ... o f_1

and

q = g_m o ... o g_1

are connected by allowed higher transformations.

This is analogous to proving equality in a type theory or simplifying words in a group presentation.

Critical-pair analysis can identify ambiguities in the rewriting system. If two local rewrite sequences lead to incompatible outcomes, the theory lacks coherence or contains a nontrivial obstruction.

Higher-dimensional rewriting extends this analysis to transformations among proofs and processes.

Automorphism detection is also essential. For each presentation x, the program should estimate or enumerate

Aut(x).

Symmetry can then be incorporated into weighting, entropy, and path integrals.

A practical computational platform should store:

objects;

typed morphisms;

composition tables;

inverse data where available;

higher cells;

cost functions;

observable values;

and provenance of each equivalence proof.

The computational representation must avoid collapsing equivalent objects too early. The transformations themselves are part of the data.

The goal is not merely to identify quotient classes. It is to preserve the internal geometry of each class.

11.8 Toy Cosmologies

A toy cosmology begins with an evolving relational complex rather than a preexisting spacetime.

At an initial stage, there may be:

a small collection of presentation types;

a set of primitive composition rules;

a coherence-cost functional;

and an update law that creates or removes relations.

The network may evolve by local moves such as:

adding a vertex;

adding or deleting an edge;

filling a coherent loop with a face;

splitting a vertex;

merging equivalent vertices;

changing an edge label;

or introducing a topological defect.

A path-integral version would sum over entire relational histories.

A deterministic version would extremize a discrete action.

A stochastic version would assign transition probabilities to local moves:

P(K -> K') proportional to exp(-Delta S/T_eff)

or, in a quantum form,

A(K -> K') proportional to exp(i Delta S/hbar).

Several questions can then be studied.

Does the network remain highly connected, fragment, or develop local neighborhoods?

Does effective dimension stabilize?

Do causal cones emerge?

Does a universal propagation speed appear?

Do defects form and persist?

Does curvature correlate with defect density?

Does the network undergo expansion?

Does entropy increase through growth of equivalent presentations?

Does a low-entropy initial condition arise naturally or require imposition?

A toy cosmology should also distinguish gauge growth from physical growth. Adding more presentations of the same underlying meaning should not automatically count as expansion of physical space.

A physical volume measure must descend through equivalence.

One possibility is to define volume by the number of inequivalent local contexts weighted by automorphism structure.

Another is to define volume spectrally through diffusion.

Another is to define it by the amount of mutually independent observable structure.

The emergence of homogeneity and isotropy can be tested statistically. Large-scale symmetry need not be imposed microscopically if many irregular networks flow toward one isotropic universality class.

Cosmological defects may correspond to nontrivial homotopy of the evolving moduli structure.

An expanding universe might be interpreted as growth in the number of coherently distinguishable relational neighborhoods rather than motion of matter into preexisting empty space.

These models would not immediately reproduce realistic cosmology. Their first purpose is to determine whether spacetime-like behavior can emerge at all from the proposed coherence dynamics.

11.9 Quantum Simulation

Finite equivalence systems can be quantized by assigning complex amplitudes to morphisms.

Let G be a finite groupoid or category of processes. Assign an action S(f) to each morphism f.

The amplitude from x to y is

K(x,y) =
sum over f:x->y
exp(iS(f)/hbar) w(f),

where w(f) includes symmetry and measure factors.

If morphisms related by higher equivalence represent the same process, the sum should be performed over the homotopy quotient.

For a finite groupoid,

K(x,y) =
sum over [f:x->y]
exp(iS(f)/hbar) / |Aut(f)|.

Composition of amplitudes should satisfy

K(x,z) =
sum_y K(y,z)K(x,y)

under suitable boundary and normalization conditions.

A finite model can therefore test whether groupoid-weighted path sums produce unitary evolution or whether additional constraints are needed.

Quantum simulation can proceed in two ways.

The first uses conventional computers to enumerate paths and sum amplitudes for small relational complexes.

The second maps the relational system onto a quantum processor. Vertices, edge labels, and coherence constraints can be encoded into qubits or qudits. Quantum evolution can then sample interference among alternative transformations directly.

A Hamiltonian may be constructed as

H =
alpha H_edge

- beta H_loop
- gamma H_defect
- delta H_constraint.

H_edge penalizes local mismatch.

H_loop measures nontrivial holonomy.

H_defect stabilizes or penalizes topological sectors.

H_constraint enforces admissible composition and equivalence.

Ground states would correspond to low-cost coherence structures. Excited states could represent defects, curvature, or broken equivalence.

The simulation should examine:

spectral gaps;

entanglement structure;

topological degeneracy;

defect mobility;

emergent gauge symmetry;

and scaling behavior.

The central test is whether familiar quantum structures emerge from equivalence dynamics rather than being inserted manually.

A model that simply encodes ordinary gauge theory in new terminology would demonstrate compatibility but not explanatory advance.

A stronger result would derive the Hilbert-space structure, probability measure, or particle sectors from the categorical architecture itself.

11.10 Parameter Estimation

A physical theory must explain or constrain numerical parameters.

Univalent Relational Dynamics introduces candidate quantities such as:

coherence penalties;

loop-obstruction coefficients;

transport costs;

automorphism volumes;

defect tensions;

and higher-cell coupling constants.

These cannot remain arbitrary if the theory is to become predictive.

One possibility is that effective parameters arise from normalization of equivalence spaces.

Suppose an interaction between sectors A and B is mediated by a morphism space

Mor(A,B).

A coupling strength might depend on the weighted volume of this space:

g_AB^2 proportional to Vol_h(Mor(A,B)).

A larger space of low-cost processes would correspond to stronger interaction.

A smaller or more constrained space would correspond to weaker interaction.

Mass may be estimated from the minimum coherence action of a defect class:

m_a c^2 =
min over E in class a
S_maint(E)/Delta t.

A topological charge may be determined by a homotopy class.

A mixing angle may arise from overlap between inequivalent decomposition bases.

A coupling ratio may arise from relative automorphism volumes.

A cosmological constant-like term may arise from residual coherence cost of the relational vacuum.

These ideas must be tested in explicit finite models.

Parameter fitting should be separated from parameter derivation.

In the fitting stage, coefficients are adjusted to reproduce known observables. This determines whether the framework is flexible enough to mimic established physics.

In the derivation stage, parameters must follow from structural constraints, normalization, consistency, or topology.

A useful theory should reduce the number of unexplained parameters rather than introduce more.

Statistical inference may compare model predictions with data. Given parameters theta and observations D,

P(theta|D) proportional to P(D|theta)P(theta).

However, the prior P(theta) should reflect the relational structure rather than arbitrary numerical convenience.

Model comparison should penalize complexity. A highly flexible coherence functional that fits any data has little explanatory value.

The research program should identify dimensionless predictions whenever possible because they are less sensitive to conventions of units and scale.

Promising targets include:

mass ratios;

coupling ratios;

topological multiplicities;

allowed symmetry groups;

dimensionality;

spectral scaling exponents;

and selection rules.

11.11 Formal Verification

The conceptual vocabulary of equivalence-first physics is vulnerable to ambiguity. Terms such as identity, equivalence, coherence, and transport can be used metaphorically unless they are formally defined.

Homotopy type theory and proof assistants provide a way to separate rigorous consequences from suggestive interpretation.

The foundational objects can be encoded as types.

Presentations are terms of a type P.

Physical equivalences are terms of

Equiv_phys(x,y).

Processes are morphisms in a typed category.

Higher coherences are identity terms between morphisms.

The univalent identity axiom becomes a formal equivalence between identity and admissible structural equivalence.

Core requirements can then be verified mechanically:

identity and inverse laws;

associativity or coherent associativity;

functoriality of observables;

descent conditions;

invariance of the action;

composition of transports;

and consistency of truncation.

A proof assistant can also detect hidden assumptions. For example, an argument may accidentally treat an equivalence as strict equality, assume a global representative exists, or use classical logic where constructive proof is required.

Formalization forces these choices to be explicit.

A staged verification program could proceed as follows.

First, formalize finite groupoid models.

Second, prove invariance of loop counts and coherence actions under relabeling.

Third, formalize the homotopy quotient and its observables.

Fourth, verify defect classification in selected finite complexes.

Fifth, formalize discrete transport and curvature.

Sixth, prove continuum-limit theorems for restricted model classes.

Formal verification will not establish that the theory describes nature. It will establish that the internal mathematical claims follow from stated assumptions.

This distinction is essential. Logical consistency and empirical truth are different requirements.

The formal library should also identify which propositions depend on univalence, choice, excluded middle, or other foundational principles.

Different foundations may yield different computational and physical interpretations.

11.12 Open Datasets and Reproducibility

A speculative foundational theory is especially vulnerable to selective examples and irreproducible claims. The computational research program must therefore be open and standardized from the beginning.

Every published model should provide:

the complete presentation set;

the admissible morphisms;

the composition rules;

the higher-cell data;

the observable definitions;

the coherence functional;

the parameter values;

the update rules;

the random seeds;

and the full numerical output.

Graph and complex data should be stored in machine-readable form.

Automorphism groups, homology, spectra, defect counts, and path-sum results should be independently checkable.

Benchmark datasets should include both positive and negative controls.

Positive controls verify that the implementation can recover known structures such as:

flat transport;

nontrivial holonomy;

simple gauge systems;

topological defects;

and standard quantum interference.

Negative controls test whether apparent emergence disappears when equivalence structure is randomized, costs are permuted, or coherence constraints are removed.

Randomized models should preserve selected low-level statistics so that comparisons are meaningful. For example, a randomized network may preserve vertex count, degree distribution, or class sizes while destroying higher coherence.

The research program should preregister key metrics when possible.

Metrics may include:

effective dimension;

spectral gap;

loop coherence;

defect stability;

Lorentz-symmetry deviation;

entropy growth;

and continuum reconstruction error.

Results should report effect sizes, confidence intervals, convergence diagnostics, and sensitivity to parameter choices.

Failed models should also be archived. Knowing which coherence actions do not produce locality, stable dimension, or causal order is scientifically valuable.

A common benchmark suite could contain:

small exactly enumerable groupoids;

simplicial defect models;

cubical transport lattices;

adaptive geometry models;

finite path-integral systems;

and toy cosmological histories.

Each benchmark should have a known expected result or clearly defined open target.

Reproducibility also requires conceptual versioning. Changes in the definition of admissible equivalence, action, or observable must be documented because they alter the theory itself, not merely its implementation.

The research program should therefore distinguish:

axiom version;

mathematical model version;

software version;

dataset version;

and parameter configuration.

The objective is to create a cumulative science rather than a sequence of disconnected metaphors.

The mathematical and computational strategy of Univalent Relational Dynamics can now be summarized.

The theory begins with an infinity-category of processes and an infinity-groupoid of identity.

Admissible equivalence is defined by preserved physical structure.

Observables are functorial and satisfy descent.

Coherence is quantified by local, loop, gluing, and higher-cell functionals.

Finite networks provide exact test systems.

Continuum physics is reconstructed through scaling, spectral analysis, and renormalization.

Homotopy-aware computation retains paths, loops, automorphisms, and higher relations.

Toy cosmologies test the emergence of dimension, locality, causality, and defect-like matter.

Quantum simulations sum amplitudes over transformation classes.

Parameter estimation asks whether masses and couplings can arise from equivalence geometry.

Formal verification separates theorem from interpretation.

Open datasets expose every construction to independent replication.

The research program will succeed only if it moves beyond philosophical restatement. It must produce explicit structures, executable algorithms, convergence results, and numerical predictions.

The foundational question is not merely whether category theory and homotopy type theory can describe physics. They already provide powerful descriptive tools.

The stronger question is:

Can a small set of equivalence and coherence principles generate the specific geometry, quantum structure, matter content, and numerical laws observed in nature?

12. Predictions, Falsifiability, Limitations, and Synthesis

12.1 Criteria for a Physical Theory

Univalent Relational Dynamics begins as a foundational proposal, but it can become a physical theory only by satisfying standards stronger than conceptual elegance. A successful reformulation must do more than provide a new vocabulary for familiar ideas. It must define mathematical structures precisely, reproduce established empirical results, identify conditions under which those results emerge, and expose itself to possible disproof.

The first requirement is internal consistency. The proposed categories, equivalence relations, transport laws, coherence functionals, probability measures, and continuum limits must be mutually compatible. It must be possible to determine which statements are definitions, which are axioms, which are derived theorems, and which remain conjectures.

The second requirement is recovery of established physics. Any proposed fundamental framework must reproduce the tested domains of classical mechanics, special relativity, general relativity, quantum mechanics, statistical mechanics, and quantum field theory within their known ranges of validity.

The third requirement is empirical distinguishability. If every measurable result is inserted into the framework as an assumption, the theory has merely translated existing physics into categorical language. A genuine theory should derive at least some structures, parameters, restrictions, or relations that are otherwise independent inputs.

The fourth requirement is computational usability. The theory must permit explicit calculation. One should be able to construct finite models, determine their equivalence classes, calculate transport costs, classify defects, evaluate amplitudes, derive effective dimensions, and compare predictions with observation.

The fifth requirement is falsifiability. The theory must identify outcomes that would count against it. Examples might include failure to recover Lorentz invariance, incorrect scaling of emergent dimension, nonunitary probability evolution, forbidden particle sectors, or predicted parameter relations inconsistent with experiment.

The sixth requirement is explanatory compression. A foundational theory should explain several phenomena through fewer principles than the theories it seeks to unify. Univalent Relational Dynamics would gain explanatory force if one coherence structure could account simultaneously for gauge symmetry, spacetime geometry, particle identity, entropy, and quantum probability.

These criteria can be summarized as

physical theory =
precise structure

- recovery of known results
- novel constraint
- calculability
- falsifiability.

A proposal lacking any of these components may remain mathematically interesting or philosophically suggestive, but it is not yet a complete physical theory.

12.2 Recovery of Gauge Theory

Gauge theory is the most immediate domain in which equivalence-first language should reproduce known physics. Gauge theory already distinguishes local presentations from invariant physical structure. The central variables are often redundant, and physically equivalent field configurations are connected by gauge transformations.

Let a local matter field be written as

phi(x),

and let a gauge transformation act as

phi(x) -> g(x) phi(x),

where g(x) belongs to a local symmetry group G.

A gauge connection A_mu specifies how internal presentations at neighboring points are compared. The covariant derivative is

D_mu phi =
partial_mu phi + A_mu phi,

with conventions depending on the representation and group.

The curvature is

F_mu_nu =
partial_mu A_nu

- partial_nu A_mu
- [A_mu,A_nu].

In Univalent Relational Dynamics, these objects receive an equivalence-first interpretation.

The matter field phi is a local presentation.

The gauge transformation g is a local self-equivalence.

The connection A_mu is a rule for transporting identity between neighboring presentations.

The curvature F_mu_nu measures failure of that transport to close trivially around infinitesimal loops.

The gauge-invariant observables are those that descend through the equivalence structure.

This reinterpretation is compatible with conventional gauge theory, but compatibility is not enough. The framework must explain why the relevant gauge groups and representations occur.

A strong result would derive an internal symmetry group as an automorphism group of a more primitive physical identity structure:

G ~= Aut(E).

Matter representations would then arise from the admissible ways stable coherence patterns transform under G.

Gauge bosons would appear as excitations of the identity-transport connection.

Charge conservation would follow from coherence under the corresponding automorphism structure.

Anomalies would appear as failures to extend classical equivalence into a consistent quantum measure.

The theory must also recover gauge fixing and ghost structure. Gauge fixing selects representatives within equivalence classes, while ghost terms compensate for changes in the measure introduced by that selection. In an equivalence-first formulation, these procedures should arise naturally from integration over a homotopy quotient rather than from a later correction to naive overcounting.

A successful recovery would show that ordinary gauge theory is a first-order or truncated approximation to a richer higher-equivalence theory.

A failed recovery would occur if the framework could not reproduce gauge-invariant amplitudes, local charge conservation, renormalizable interactions, or the known relation between connection and curvature.

Gauge theory is therefore both the easiest test and a severe one. Because its equivalence structure is already well developed, any new framework must reproduce it without ambiguity and then provide additional explanatory value.

12.3 Recovery of General Relativity

General relativity describes spacetime geometry through a metric tensor g_mu_nu and its associated curvature. Matter and energy influence geometry through the Einstein field equation:

G_mu_nu + Lambda g_mu_nu = kappa T_mu_nu.

Univalent Relational Dynamics proposes that the metric is an effective summary of coherence cost and that curvature is failed identity closure. To recover general relativity, this interpretation must be made quantitative.

The first task is to derive an effective metric from transport cost. If x and x + dx are neighboring relational contexts, one seeks a local expansion

C(x,x+dx)^2 =
g_mu_nu(x) dx^mu dx^nu

- higher-order terms.

The second task is to show that extremal coherence transport yields the geodesic equation:

d^2 x^mu/dtau^2

- Gamma^mu_(alpha beta)
  (dx^alpha/dtau)(dx^beta/dtau)
  = 0.

The third task is to derive a curvature tensor from loop transport. The composition of infinitesimal transports should produce the Riemann curvature tensor or an equivalent connection-based structure.

The fourth task is to identify a relational action whose continuum limit contains the Einstein-Hilbert term:

S_grav =
(1/2kappa)
integral
(R - 2Lambda)
sqrt(-g) d^4x.

The fifth task is to show how stable coherence patterns generate an effective stress-energy tensor T_mu_nu.

The sixth task is to derive the equality of inertial and gravitational mass. If both are expressions of the same coherence-maintenance functional, the equivalence principle may emerge structurally.

Recovery must include more than formal resemblance. The theory should reproduce:

gravitational redshift;

light bending;

orbital precession;

time dilation;

gravitational waves;

black-hole exterior geometry;

and cosmological expansion within tested regimes.

The theory must also explain why the effective geometry is Lorentzian rather than Euclidean and why it possesses one time-like dimension.

A particularly important question is whether background independence is genuine. If the relational network is placed secretly inside a preexisting spacetime, the theory has not derived geometry. It has only redescribed fields on a background.

The stronger program requires spacetime points, dimension, metric, and causal structure to emerge from the pattern of admissible processes and equivalence costs.

A successful result would derive Einstein-like dynamics as a large-scale equation of state or coherence condition.

A failure would occur if no stable Lorentzian continuum emerged, if the effective gravitational degrees of freedom were wrong, or if unavoidable deviations conflicted with precision tests.

12.4 Recovery of Quantum Mechanics

The quantum sector imposes several non-negotiable requirements.

The theory must recover complex amplitudes, interference, unitary evolution where appropriate, tensor composition, entanglement, contextuality, measurement statistics, and the Born rule.

A candidate relational amplitude has the form

K([x],[y]) =
integral over e:x->y
exp(iS[e]/hbar) Dmu(e).

For this expression to reproduce quantum mechanics, the space of transformations and the measure Dmu must support coherent addition of alternatives.

The theory must explain why amplitudes are complex rather than ordinary probabilities.

It must explain why composition is linear.

It must explain why reversible evolution is represented by unitary transformations.

It must explain why independent systems combine through tensor products.

It must explain why nonfactorizable composites exhibit entanglement.

Most importantly, it must recover

P(i) = |c_i|^2.

The Born rule cannot be assumed merely because the target theory uses it. A foundational derivation should show why the measure on truncated outcome sectors is the squared magnitude of a complex relational amplitude.

One possible route is that probability arises from pairing a transformation with its conjugate:

P(i) =
A_i A_i*.

Another is that consistency under composition, symmetry, and additivity uniquely selects the quadratic norm.

Another is that the quantum measure is a homotopy-weighted volume on paired process spaces.

Whatever route is chosen, it must be explicit and noncircular.

The measurement problem also remains central. Truncation and decoherence explain why higher paths become inaccessible locally, but they do not automatically explain why a single outcome is experienced.

The theory must state whether it adopts:

objective selection;

relative-state branching;

relational actualization;

stochastic collapse;

or another mechanism.

A physical theory must also preserve no-signaling and reproduce Bell-inequality violations without permitting controllable superluminal communication.

Contextuality should emerge from a failure of local classical value assignments to glue globally.

The theory must further recover quantum field theory, where particle number changes and fields possess infinitely many degrees of freedom.

A successful recovery would show that Hilbert-space quantum mechanics is a representation of an underlying process category and that its linear structure follows from the composition of equivalence amplitudes.

A failure would occur if amplitudes could not be normalized, if evolution became nonunitary without empirical justification, or if the Born rule required independent insertion.

12.5 Novel Predictions

A new foundational framework becomes scientifically valuable when it predicts more than the theories it reproduces.

Univalent Relational Dynamics suggests several possible classes of prediction, although none is established merely by the conceptual framework.

The first class concerns symmetry restriction. If gauge groups arise as automorphism groups of admissible identity structures, not every mathematically possible group may occur. The theory could predict a restricted family of internal symmetries or representations.

The second class concerns particle classification. If particles are stable coherence defects, the homotopy structure of the underlying moduli object may permit only specific defect classes.

This could constrain the number of particle families, allowed charges, or possible stable exotic particles.

The third class concerns mass relations. If rest mass is the minimum maintenance cost of a coherence class, ratios of masses might follow from topological or categorical invariants rather than independent parameters.

The fourth class concerns coupling constants. Interaction strengths may depend on weighted volumes of morphism spaces:

g_AB^2 proportional to Vol_h(Mor(A,B)).

Such a relation could produce dimensionless coupling ratios.

The fifth class concerns spacetime dimension. A coherence-extremization principle may favor a stable effective dimension. If four-dimensional Lorentzian geometry appears as a unique or dominant phase, dimensionality would become a prediction rather than an input.

The sixth class concerns high-energy deviations from locality or Lorentz invariance. A discrete or network-like microscopic structure may produce modified dispersion relations:

E^2 =
p^2 c^2

- m^2 c^4
- eta p^3/M
- ...

The absence of such corrections below observed limits would constrain the microscopic model.

The seventh class concerns entropy corrections. Homotopy cardinality may modify conventional state counting when automorphism groups are large. This could affect highly symmetric phases, topological systems, or black-hole entropy.

The eighth class concerns quantum interference among topologically distinct process classes. Additional phase sectors might arise from higher morphisms or nontrivial automorphisms.

The ninth class concerns cosmology. Growth of relational volume, defect formation, or changes in effective dimension could leave signatures in primordial fluctuations, relic backgrounds, or large-scale topology.

The tenth class concerns forbidden processes. If no admissible morphism exists between two identity sectors, the corresponding interaction is exactly prohibited. Selection rules could therefore arise categorically.

These possibilities should be treated as research targets, not as predictions already obtained.

A prediction becomes scientific only when the theory supplies:

a specific mathematical model;

a quantitative value or inequality;

an identified experimental observable;

and a result that could disagree with measurement.

Without these components, references to new particles, dimensions, or couplings remain speculative.

12.6 Distinguishing the Framework from Relabeling

Category theory, homotopy theory, gauge theory, and relational interpretations already exist within modern mathematical physics. A serious objection is that Univalent Relational Dynamics may simply rename familiar concepts.

Gauge transformations become equivalences.

Connections become identity transports.

Curvature becomes coherence failure.

Particles become defects.

Charge becomes automorphism response.

Measurement becomes truncation.

If these substitutions leave all equations and explanatory dependencies unchanged, the framework is a change of language rather than a new theory.

A genuine advance must occur at one or more of four levels.

The first is derivation. The framework must derive structures that conventional theories assume. Examples include spacetime dimension, gauge group, particle spectrum, or probability rule.

The second is unification. One mathematical principle should account for phenomena currently described by separate mechanisms. For example, the same coherence obstruction might generate both gauge curvature and material defect structure.

The third is restriction. The framework should rule out theories that conventional mathematics allows. It may select only certain categories, automorphism groups, actions, or continuum limits.

The fourth is prediction. It should produce measurable relations not inserted as assumptions.

A useful diagnostic is to compare the dependency graphs of the old and new theories.

If general relativity assumes a metric and the new theory derives that metric from transport cost, explanatory structure has changed.

If quantum theory assumes Hilbert space and the new theory derives it from process composition, explanatory structure has changed.

If the new theory assumes a metric, Hilbert space, gauge group, and particle spectrum in order to reproduce them, it has not gained explanatory depth.

The framework must also avoid replacing one unexplained object with another. A coherence functional with arbitrary coefficients can encode any desired physics. Unless those coefficients are constrained, the formalism has merely moved the freedom into a new location.

The standard of success is therefore severe:

The theory must make known structure less arbitrary.

The phrase equivalence-first should correspond to a reduction in primitive assumptions, not merely a preference for relational language.

12.7 Mathematical Risks

The mathematical program faces substantial difficulties.

The first risk is foundational ambiguity. There are many inequivalent models of higher categories, infinity-groupoids, higher topoi, and derived geometry. A physical claim may depend on technical choices that appear philosophically similar but are mathematically distinct.

The second risk is absence of a suitable measure. Path integration over ordinary fields is already difficult to define rigorously. Integration over higher categories, homotopy quotients, and variable relational complexes is harder.

The third risk is overcounting or undercounting equivalence classes. Automorphisms, stabilizers, and higher symmetries affect weights. Incorrect treatment can change amplitudes and entropy.

The fourth risk is continuum failure. A discrete coherence network may not produce a smooth manifold, stable dimension, local field theory, or Lorentz invariance.

The fifth risk is uncontrolled nonlocality. Equivalence relations can connect distant presentations. Unless local structure emerges dynamically, the theory may permit interactions incompatible with observed causal behavior.

The sixth risk is trivialization. Maximizing equivalence may drive the system toward a featureless phase in which all presentations collapse into one meaning.

The seventh risk is fragmentation. Penalizing mismatch too strongly may break the network into disconnected sectors with no large-scale geometry.

The eighth risk is dependence on arbitrary presentation. A proposed coherence cost may change under relabeling or gauge transformation, making it nonphysical.

The ninth risk is failure of strict composition. Higher categories often satisfy associativity only up to coherent equivalence. Computational implementations must retain this structure rather than replacing it casually with exact equality.

The tenth risk is undecidability or intractability. Determining equivalence in a rich higher category may be computationally impossible for general systems.

The eleventh risk is singular moduli structure. Quotient spaces often contain points with enhanced symmetry, non-Hausdorff behavior, or derived obstructions. These may be physically meaningful but mathematically difficult.

The twelfth risk is absence of uniqueness. Many coherence actions may reproduce the same low-energy physics. The framework then fails to select a fundamental model.

These risks do not invalidate the program, but they define the burden of proof. Every broad conceptual claim must eventually be translated into a model where these issues can be evaluated directly.

12.8 Philosophical Risks

The framework also faces philosophical dangers.

The first is the reification of mathematics. A useful mathematical representation does not automatically reveal ontology. The success of category theory in organizing physical models does not prove that reality literally is a category.

The second is the conflation of equivalence with identity. Two structures may be equivalent for one purpose and different for another. Physical identity must always specify which structures and observables are preserved.

The third is observer relativism. Because presentations depend on context, one may incorrectly conclude that reality is subjective. The intended claim is instead that objective reality is encoded in the transformations and invariants connecting contexts.

The fourth is elimination of genuine difference. An equivalence-first ontology must account for inequivalent sectors, irreversible processes, and physically meaningful asymmetries. It cannot reduce everything to sameness.

The fifth is explanatory circularity. If equivalence is defined as whatever preserves physical reality, while physical reality is defined as whatever equivalence preserves, the theory becomes vacuous. Admissibility must be specified independently through formal structure and empirical content.

The sixth is misuse of univalence. Mathematical univalence concerns equivalence of types within a formal foundation. Its physical translation requires justification. One cannot simply replace mathematical identity with physical identity without defining the relevant physical universe of structures.

The seventh is category error between ontology and epistemology. Different descriptions may reflect limited knowledge rather than multiple presentations of one physical entity. The theory must distinguish descriptive ignorance from genuine gauge redundancy.

The eighth is excessive metaphor. Terms such as coherence, path, identity, and obstruction have precise meanings in mathematics but looser meanings in philosophy. The paper's claims must be assessed only after those meanings are formalized.

The ninth is aesthetic overconfidence. Elegant unification is not evidence by itself. Nature is not required to use the structures humans find conceptually satisfying.

The tenth is simulation speculation. A finite code-like model may tempt claims that the universe is programmed or encrypted. Such claims do not follow from equivalence geometry.

The strongest philosophical position is therefore restrained.

Univalent Relational Dynamics should be treated as a proposal for organizing and possibly deriving physical structure, not as proof that mathematical equivalence is the final metaphysical substance of reality.

12.9 The Role of Finite Analogies

Finite analogies are indispensable during early theory development. They permit exact enumeration, complete visualization, and rigorous comparison among candidate structures.

The genetic-code toy model illustrates presentations, quotient meanings, local identity-preserving paths, rank deficiency, and an extremal geometry. Other useful finite models may include:

group actions on finite sets;

lattice gauge systems;

spin networks;

cellular automata;

finite groupoids;

simplicial complexes;

tensor networks;

and error-correcting codes.

The value of these systems lies in structural transparency.

One can calculate every equivalence class.

One can enumerate all automorphisms.

One can measure loop coherence.

One can test alternative geometries.

One can randomize the interpretation map.

One can determine whether an extremum is unique.

However, analogies have strict limits.

A finite graph does not automatically reproduce continuity.

A quotient map does not automatically imply quantum measurement.

A loop does not automatically represent curvature.

A rank deficiency does not automatically represent hidden physical dimension.

A code does not automatically imply a programmer.

The legitimate use of analogy is to isolate transferable mathematical principles.

The illegitimate use is to infer a shared origin or ontology from a superficial structural resemblance.

A finite model contributes to the research program when it supports a theorem of the form:

Under assumptions A, B, and C, the system necessarily exhibits property D.

It contributes less when it supports only a suggestive comparison.

The genetic-code model therefore functions best as an existence proof that many presentations, fewer meanings, and geometry-dependent equivalence compactness can coexist in one finite system.

It does not establish that fundamental physics is encoded biologically.

The broader transferable hypothesis is that a physical geometry may be selected by localization of equivalence.

That hypothesis must be tested in nonbiological systems before any foundational significance is assigned.

12.10 A Staged Research Strategy

The development of Univalent Relational Dynamics should proceed in stages.

The first stage is finite formalization.

Construct small groupoids and higher complexes with explicit presentation spaces, equivalence maps, automorphisms, and coherence actions. Verify all calculations exactly.

The second stage is geometry emergence.

Define transport costs on finite networks and determine whether effective topology, metric, dimension, and curvature can be reconstructed without assuming a manifold.

The third stage is defect matter.

Classify stable topological and categorical obstructions. Determine whether localized excitations behave like persistent particles under transport and interaction.

The fourth stage is quantum amplitude.

Assign complex weights to process classes. Test interference, composition, normalization, and groupoid weighting.

The fifth stage is probability.

Derive or constrain the Born rule from the measure and composition of equivalence amplitudes.

The sixth stage is causal structure.

Introduce directed noninvertible morphisms and test the emergence of partial order, finite signal propagation, and temporal asymmetry.

The seventh stage is continuum limit.

Show that families of finite models converge toward local field equations and smooth transport structures.

The eighth stage is gauge and gravitational recovery.

Derive connection, curvature, gauge symmetry, geodesic motion, and Einstein-like equations in controlled limits.

The ninth stage is parameter constraint.

Determine whether masses, couplings, dimensions, or symmetry groups follow from the relational structure.

The tenth stage is empirical comparison.

Identify precision experiments capable of distinguishing the theory from established models.

Each stage should have a failure condition.

If no stable metric emerges, the geometry program fails.

If defect spectra are arbitrary, the matter program fails.

If amplitudes do not compose unitarily, the quantum program fails.

If the Born rule must be inserted, the foundational quantum claim remains incomplete.

If Lorentz invariance does not emerge, the relativistic program fails.

If no novel parameter relation appears, the framework may remain a reformulation.

This staged strategy prevents the theory from moving directly from philosophical premise to cosmological conclusion.

It replaces broad speculation with a sequence of tractable mathematical tests.

12.11 Unified Interpretation

The central proposal can now be assembled into one structure.

A presentation is a local way reality appears.

An equivalence is a transformation preserving physical identity.

A higher equivalence relates alternative transformations.

A process is a directed morphism that may alter physical identity.

An observable is a construction that descends through admissible equivalence.

A geometry is a large-scale organization of transport cost.

Curvature is the failure of identity transport to close trivially.

Matter is a stable coherence pattern or obstruction.

Mass is the cost of maintaining and transporting that pattern.

Charge is its transformation under automorphisms.

Interaction is a change in how coherence patterns compose.

Entanglement is failure of composite identity to factor.

Measurement is truncation of accessible higher relation.

Entropy is the multiplicity of fine presentations corresponding to one coarse meaning.

Time is directed composition of noninvertible process.

The arrow of time is the direction of increasing relational multiplicity and stable record formation.

These interpretations are not independent metaphors. They are intended to arise from one mathematical architecture.

Let R denote a higher category of physical presentations and processes.

Its invertible core

Core(R)

is the infinity-groupoid of physical identity.

Its noninvertible morphisms encode change and causation.

Its automorphisms encode symmetry.

Its homotopy groups encode stable sectors and defects.

Its local transport maps encode connection.

Its loop compositions encode curvature.

Its truncation functors encode observation and coarse-graining.

Its weighted morphism spaces encode amplitudes and interaction strengths.

Its moduli fibers encode entropy.

The central ambition is therefore unification through structural economy.

Rather than adding separate ontologies for matter, spacetime, quantum state, and information, the theory seeks to derive them as different manifestations of equivalence and its obstruction.

This unification would be significant only if the same formal structure produces the correct quantitative laws in each domain.

The framework should not merely say that everything is relational. It must specify which relations exist, how they compose, what they cost, how they are measured, and why their large-scale behavior matches observation.

12.12 Final Conclusion

Modern physics repeatedly teaches that mathematical presentation is not physical reality.

Coordinates are not spacetime.

Gauge potentials are not uniquely observable fields.

Quantum vectors are not uniquely physical rays.

Microscopic configurations are not macroscopic identities.

Local descriptions are not complete global structures.

Yet physical theories often continue to begin with states and treat equivalence as a secondary correction.

Univalent Relational Dynamics reverses that order.

Its foundational claim is:

Reality is not made of states; it is made of equivalence among states.

This claim does not abolish states. It relocates them.

A state is a local presentation.

Its identity is constituted by the admissible transformations connecting it to other presentations.

Its persistence is the coherent transport of that identity.

Its symmetry is its automorphism structure.

Its geometry is the cost of carrying it.

Its curvature is the obstruction to returning unchanged.

Its materiality is the stability of that obstruction.

Its quantum behavior is the interference among alternative transformations.

Its classical appearance is the truncation of higher paths.

Its entropy is the multiplicity of presentations that observation no longer distinguishes.

The complete physical object is therefore not a point in a state space. It is a homotopy-coherent pattern within a category of processes.

The theory's deepest ontological statement can be written as

physical entity =
persistent equivalence structure
under admissible transformation.

The deepest dynamical statement is

physical law =
rule governing the transport, composition, cost, and obstruction of equivalence.

The deepest geometric statement is

distance =
minimum cost of coherent identification.

The deepest quantum statement is

amplitude =
weighted sum over admissible transformations.

The deepest thermodynamic statement is

entropy =
measure of equivalent presentation under truncation.

The genetic-code toy model illustrates a finite version of this architecture: many symbolic presentations map to fewer meanings, local transformations sometimes preserve those meanings, and a preferred coordinate geometry can maximize the locality of equivalence. The physical theory must move far beyond that analogy, but the finite example demonstrates the logical possibility of a world organized less by individual states than by the paths connecting states of equal meaning.

The ultimate scientific question is not whether category theory and homotopy type theory can describe existing physics. They clearly can describe important aspects of symmetry, topology, process, and identity.

The stronger question is whether equivalence can generate physics.

Can spacetime emerge from coherence cost?

Can curvature emerge from failed identity closure?

Can matter emerge from stable obstruction?

Can charge emerge from automorphism response?

Can quantum probability emerge from measures on process spaces?

Can entropy emerge from homotopy-aware presentation multiplicity?

Can the known constants and particle classes be derived from one relational architecture?

Until these questions receive quantitative answers, Univalent Relational Dynamics remains a research program.

Its value lies in changing the foundational problem.

The conventional question asks:

What are the fundamental states of the universe?

The equivalence-first question asks:

What allows different presentations to count as the same reality, which obstructions prevent that identification, and what world is generated by the resulting pattern of sameness and difference?

The proposed answer is that objects, geometry, matter, and law may all be secondary.

What is primary is the structured possibility of remaining the same through change.

Reality is not made of states.

Reality is made of the paths that allow states to count as the same.

13. References

1. Youvan, D. C. The Genetic Code as Physicochemical and Topological Geometry: Nucleotide-Determined Hydropathy, Exact Pseudoinverse Mapping, and the G-A-C-U Maximum. ResearchGate preprint. 2026. DOI: 10.13140/RG.2.2.25956.95366.
2. Yang, M. M., Coleman, W. J., and Youvan, D. C. Genetic coding algorithms for engineering membrane proteins. In: Michel-Beyerle, M. E., editor. Reaction Centers of Photosynthetic Bacteria. Springer Series in Biophysics, Volume 6. Berlin and Heidelberg: Springer-Verlag; 1990. pp. 209–218. DOI: 10.1007/978-3-642-61297-8_21.
3. Youvan, D. C., and Juvon, C. Decryption and Topology of the Genetic Code. ResearchGate preprint. 2022. DOI: 10.13140/RG.2.2.34261.91361.
4. Youvan, D. C. Graph Analysis of the Genetic Code Using Closed-Loops Shows Isotropic Axes Using the Order GACU Yields Maximum Topological Compactness. ResearchGate preprint. 2022. DOI: 10.13140/RG.2.2.21318.55362.
5. Youvan, D. C. The Genetic Code Has a Formulae-Based Pseudo-Inverse. ResearchGate preprint. 2023. DOI: 10.13140/RG.2.2.18540.03203.
6. Eilenberg, S., and Mac Lane, S. General theory of natural equivalences. Transactions of the American Mathematical Society. 1945;58(2):231–294. DOI: 10.1090/S0002-9947-1945-0013131-6.
7. Mac Lane, S. Categories for the Working Mathematician. Second edition. Graduate Texts in Mathematics, Volume 5. New York: Springer; 1998.
8. Awodey, S. Category Theory. Second edition. Oxford Logic Guides, Volume 52. Oxford: Oxford University Press; 2010.
9. Leinster, T. Higher Operads, Higher Categories. London Mathematical Society Lecture Note Series, Volume 298. Cambridge: Cambridge University Press; 2004.
10. Baez, J. C., and Dolan, J. Higher-dimensional algebra and topological quantum field theory. Journal of Mathematical Physics. 1995;36(11):6073–6105. DOI: 10.1063/1.531236.
11. Lurie, J. Higher Topos Theory. Annals of Mathematics Studies, Volume 170. Princeton: Princeton University Press; 2009.
12. Lurie, J. Higher Algebra. Unpublished manuscript. 2017.

13. Rezk, C. A model for the homotopy theory of homotopy theory. Transactions of the American Mathematical Society. 2001;353(3):973–1007. DOI: 10.1090/S0002-9947-00-02653-2.

14. Joyal, A., and Tierney, M. Quasi-categories versus Segal spaces. In: Kock, J., editor. Categories in Algebra, Geometry and Mathematical Physics. Contemporary Mathematics, Volume 431. Providence: American Mathematical Society; 2007. pp. 277–326.

15. The Univalent Foundations Program. Homotopy Type Theory: Univalent Foundations of Mathematics. Princeton: Institute for Advanced Study; 2013.

16. Voevodsky, V. Univalent foundations of mathematics. In: Beklemishev, L. D., and de Queiroz, R., editors. Logic, Language, Information and Computation. Lecture Notes in Computer Science, Volume 6642. Berlin and Heidelberg: Springer; 2011. pp. 4–4. DOI: 10.1007/978-3-642-20920-8_4.

17. Awodey, S., and Warren, M. A. Homotopy-theoretic models of identity types. Mathematical Proceedings of the Cambridge Philosophical Society. 2009;146(1):45–55. DOI: 10.1017/S0305004108001783.

18. Kapulkin, K., Lumsdaine, P. L., and Voevodsky, V. The simplicial model of univalent foundations. Journal of the European Mathematical Society. 2021;23(6):2071–2126. DOI: 10.4171/JEMS/1050.

19. Ahrens, B., Kapulkin, K., and Shulman, M. Univalent categories and the Rezk completion. Mathematical Structures in Computer Science. 2015;25(5):1010–1039. DOI: 10.1017/S0960129514000486.

20. Hofmann, M., and Streicher, T. The groupoid interpretation of type theory. In: Sambin, G., and Smith, J. M., editors. Twenty-Five Years of Constructive Type Theory. Oxford Logic Guides, Volume 36. Oxford: Oxford University Press; 1998. pp. 83–111.

21. Martin-Löf, P. Intuitionistic Type Theory. Naples: Bibliopolis; 1984.

22. Baez, J. C., and Shulman, M. Lectures on n-categories and cohomology. In: Baez, J. C., and May, J. P., editors. Towards Higher Categories. The IMA Volumes in Mathematics and its Applications, Volume 152. New York: Springer; 2010. pp. 1–68.

23. Baez, J. C., and Stay, M. Physics, topology, logic and computation: A Rosetta Stone. In: Coecke, B., editor. New Structures for Physics. Lecture Notes in Physics, Volume 813. Berlin and Heidelberg: Springer; 2011. pp. 95–172.

24. Abramsky, S., and Coecke, B. A categorical semantics of quantum protocols. Proceedings of the 19th Annual IEEE Symposium on Logic in Computer Science. Los Alamitos: IEEE Computer Society; 2004. pp. 415–425. DOI: 10.1109/LICS.2004.1319636.

25. Abramsky, S., and Coecke, B. Categorical quantum mechanics. In: Engesser, K., Gabbay, D. M., and Lehmann, D., editors. Handbook of Quantum Logic and Quantum Structures: Quantum Logic. Amsterdam: Elsevier; 2009. pp. 261–323.

26. Coecke, B., and Kissinger, A. Picturing Quantum Processes: A First Course in Quantum Theory and Diagrammatic Reasoning. Cambridge: Cambridge University Press; 2017.

27. Heunen, C., and Vicary, J. Categories for Quantum Theory: An Introduction. Oxford: Oxford University Press; 2019.

28. Selinger, P. A survey of graphical languages for monoidal categories. In: Coecke, B., editor. New Structures for Physics. Lecture Notes in Physics, Volume 813. Berlin and Heidelberg: Springer; 2011. pp. 289–355.

29. Atiyah, M. F. Topological quantum field theories. Publications Mathématiques de l'Institut des Hautes Études Scientifiques. 1988;68:175–186. DOI: 10.1007/BF02698547.

30. Witten, E. Quantum field theory and the Jones polynomial. Communications in Mathematical Physics. 1989;121:351–399. DOI: 10.1007/BF01217730.

31. Baez, J. C., and Huerta, J. An invitation to higher gauge theory. General Relativity and Gravitation. 2011;43:2335–2392. DOI: 10.1007/s10714-010-1070-9.

32. Baez, J. C., and Schreiber, U. Higher gauge theory. In: Kock, J., editor. Categories in Algebra, Geometry and Mathematical Physics. Contemporary Mathematics, Volume 431. Providence: American Mathematical Society; 2007. pp. 7–30.

33. Baez, J. C., and Wise, D. K. Teleparallel gravity as a higher gauge theory. Communications in Mathematical Physics. 2015;333(1):153–186. DOI: 10.1007/s00220-014-2178-7.

34. Rovelli, C. Relational quantum mechanics. International Journal of Theoretical Physics. 1996;35(8):1637–1678. DOI: 10.1007/BF02302261.

35. Oeckl, R. A general boundary formulation for quantum mechanics and quantum gravity. Physics Letters B. 2003;575(3–4):318–324. DOI: 10.1016/j.physletb.2003.08.043.

36. Oeckl, R. General boundary quantum field theory: Foundations and probability interpretation. Advances in Theoretical and Mathematical Physics. 2008;12(2):319–352. DOI: 10.4310/ATMP.2008.v12.n2.a3.

37. Markopoulou, F. Quantum causal histories. Classical and Quantum Gravity. 2000;17(10):2059–2072. DOI: 10.1088/0264-9381/17/10/303.

38. Hardy, L. Towards quantum gravity: A framework for probabilistic theories with non-fixed causal structure. Journal of Physics A: Mathematical and Theoretical. 2007;40(12):3081–3099. DOI: 10.1088/1751-8113/40/12/S12.

39. Bombelli, L., Lee, J., Meyer, D., and Sorkin, R. D. Space-time as a causal set. Physical Review Letters. 1987;59(5):521–524. DOI: 10.1103/PhysRevLett.59.521.

40. Malament, D. B. The class of continuous timelike curves determines the topology of spacetime. Journal of Mathematical Physics. 1977;18(7):1399–1404. DOI: 10.1063/1.523436.

41. Einstein, A. Die Grundlage der allgemeinen Relativitätstheorie. Annalen der Physik. 1916;49:769–822.

42. Misner, C. W., Thorne, K. S., and Wheeler, J. A. Gravitation. San Francisco: W. H. Freeman; 1973.

43. Wald, R. M. General Relativity. Chicago: University of Chicago Press; 1984.

44. Jacobson, T. Thermodynamics of spacetime: The Einstein equation of state. Physical Review Letters. 1995;75(7):1260–1263. DOI: 10.1103/PhysRevLett.75.1260.

45. Yang, C. N., and Mills, R. L. Conservation of isotopic spin and isotopic gauge invariance. Physical Review. 1954;96(1):191–195. DOI: 10.1103/PhysRev.96.191.

46. Noether, E. Invariant variation problems. Transport Theory and Statistical Physics. 1971;1(3):186–207. English translation of the 1918 paper. DOI: 10.1080/00411457108231446.

47. Bell, J. S. On the Einstein-Podolsky-Rosen paradox. Physics Physique Fizika. 1964;1(3):195–200. DOI: 10.1103/PhysicsPhysiqueFizika.1.195.

48. Kochen, S., and Specker, E. P. The problem of hidden variables in quantum mechanics. Journal of Mathematics and Mechanics. 1967;17(1):59–87. DOI: 10.1512/iumj.1968.17.17004.

49. Gleason, A. M. Measures on the closed subspaces of a Hilbert space. Journal of Mathematics and Mechanics. 1957;6(4):885–893. DOI: 10.1512/iumj.1957.6.56050.

50. Zurek, W. H. Decoherence, einselection, and the quantum origins of the classical. Reviews of Modern Physics. 2003;75(3):715–775. DOI: 10.1103/RevModPhys.75.715.

51. Everett, H. Relative state formulation of quantum mechanics. Reviews of Modern Physics. 1957;29(3):454–462. DOI: 10.1103/RevModPhys.29.454.

52. von Neumann, J. Mathematical Foundations of Quantum Mechanics. Princeton: Princeton University Press; 1955.

53. Feynman, R. P., and Hibbs, A. R. Quantum Mechanics and Path Integrals. New York: McGraw-Hill; 1965.

54. Dirac, P. A. M. The Principles of Quantum Mechanics. Fourth edition. Oxford: Clarendon Press; 1958.

55. Ryu, S., and Takayanagi, T. Holographic derivation of entanglement entropy from the anti-de Sitter space/conformal field theory correspondence. Physical Review Letters. 2006;96(18):181602. DOI: 10.1103/PhysRevLett.96.181602.

56. Van Raamsdonk, M. Building up spacetime with quantum entanglement. General Relativity and Gravitation. 2010;42:2323–2329. DOI: 10.1007/s10714-010-1034-0.

57. Bekenstein, J. D. Black holes and entropy. Physical Review D. 1973;7(8):2333–2346. DOI: 10.1103/PhysRevD.7.2333.

58. Hawking, S. W. Particle creation by black holes. Communications in Mathematical Physics. 1975;43:199–220. DOI: 10.1007/BF02345020.

59. Shannon, C. E. A mathematical theory of communication. Bell System Technical Journal. 1948;27:379–423 and 623–656. DOI: 10.1002/j.1538-7305.1948.tb01338.x.

60. Jaynes, E. T. Information theory and statistical mechanics. Physical Review. 1957;106(4):620–630. DOI: 10.1103/PhysRev.106.620.

61. Baez, J. C., Fritz, T., and Leinster, T. A characterization of entropy in terms of information loss. Entropy. 2011;13(11):1945–1957. DOI: 10.3390/e13111945.

62. Baez, J. C., Hoffnung, A. E., and Walker, C. D. Higher-dimensional algebra VII: Groupoidification. Theory and Applications of Categories. 2010;24:489–553.

63. Weinstein, A. Symplectic groupoids and Poisson manifolds. Bulletin of the American Mathematical Society. 1987;16(1):101–104. DOI: 10.1090/S0273-0979-1987-15473-5.

64. Kock, J. Frobenius Algebras and 2-D Topological Quantum Field Theories. London Mathematical Society Student Texts, Volume 59. Cambridge: Cambridge University Press; 2003.

65. Costello, K. Renormalization and Effective Field Theory. Mathematical Surveys and Monographs, Volume 170. Providence: American Mathematical Society; 2011.

66. Mermin, N. D. The topological theory of defects in ordered media. Reviews of Modern Physics. 1979;51(3):591–648. DOI: 10.1103/RevModPhys.51.591.

67. Kibble, T. W. B. Topology of cosmic domains and strings. Journal of Physics A: Mathematical and General. 1976;9(8):1387–1398. DOI: 10.1088/0305-4470/9/8/029.

68. Nakahara, M. Geometry, Topology and Physics. Second edition. Boca Raton: CRC Press; 2003.

69. Hamming, R. W. Error detecting and error correcting codes. Bell System Technical Journal. 1950;29(2):147–160. DOI: 10.1002/j.1538-7305.1950.tb00463.x.

70. Moore, E. H. On the reciprocal of the general algebraic matrix. Bulletin of the American Mathematical Society. 1920;26:394–395.

71. Penrose, R. A generalized inverse for matrices. Proceedings of the Cambridge Philosophical Society. 1955;51:406–413. DOI: 10.1017/S0305004100030401.

72. Golub, G. H., and Reinsch, C. Singular value decomposition and least squares solutions. Numerische Mathematik. 1970;14:403–420. DOI: 10.1007/BF02163027.

73. Edelsbrunner, H., and Harer, J. Computational Topology: An Introduction. Providence: American Mathematical Society; 2010.

74. Carlsson, G. Topology and data. Bulletin of the American Mathematical Society. 2009;46(2):255–308. DOI: 10.1090/S0273-0979-09-01249-X.

75. Ollivier, Y. Ricci curvature of Markov chains on metric spaces. Journal of Functional Analysis. 2009;256(3):810–864. DOI: 10.1016/j.jfa.2008.11.001.

76. Kyte, J., and Doolittle, R. F. A simple method for displaying the hydropathic character of a protein. Journal of Molecular Biology. 1982;157(1):105–132. DOI: 10.1016/0022-2836(82)90515-0.

77. Zamyatnin, A. A. Protein volume in solution. Progress in Biophysics and Molecular Biology. 1972;24:107–123. DOI: 10.1016/0079-6107(72)90005-3.

78. Ladyman, J., and Ross, D., with Spurrett, D., and Collier, J. Every Thing Must Go: Metaphysics Naturalized. Oxford: Oxford University Press; 2007.

79. French, S. The Structure of the World: Metaphysics and Representation. Oxford: Oxford University Press; 2014.

80. Wheeler, J. A. Law without law. In: Wheeler, J. A., and Zurek, W. H., editors. Quantum Theory and Measurement. Princeton: Princeton University Press; 1983. pp. 182–213.

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